A FIRST BOOK
IN MUSIC

A FIRST BOOK IN LOGIC

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PREFACE

The recent developments in logical theory following upon the contributions of Boole, Pierce, and Schröder have seemed to place the subject beyond the reach of the average student and even in most instances beyond the reach of the technically equipped philosopher. The result has been to reduce the attention that was formerly given to elementary instruction in logic and to displace the traditional course from its originally dominant position in the university curriculum. The conviction is abroad that the ancient *organon* is so far inferior to the modern instrument perfected by the critical labors of Peano, Frege, Russell, and others, that it no longer deserves the attention once bestowed upon it. These objections the writer has endeavored to meet, in the first instance by introducing no symbols whatever, save the ones employed by traditional logic itself, so that the treatment may be followed by any intelligent reader, and secondly by keeping modern developments always in mind while following the traditional order of treatment. Finally, he has met the recent contention that the classical system does not hold true in all of its parts, by showing in the last chapter of this work that

this view is based upon a misapprehension. In completing the chapters dealing with the theoretical portion of the subject the writer has availed himself of parts of his *Letters on Logic,* which in turn is based upon a brief syllabus which Professor Singer once placed at his disposal, and also of parts of his recently published *Foundations of Formal Logic.*

H. B. S.

TERMS, RELATIONS, PROPOSITIONS

CHAPTER I

INTRODUCTION TO INFERENCE

Every deductive science is concerned with a set of **relationships** which are peculiar to its domain or field of application, and a set of **objects** of which it is meaningful to say that they stand in these relationships to one another. Suppose a set of objects, *a, b, c,* etc., which stand for classes or groups of things, and the relation of **inclusion,** and consider the statement,

a is included in *b*.

If *a* stands for *Athenians* and *b* stands for *Greeks,* it is clear that the members of the *a* class are contained among the members of the *b* class and the statement that *a* is included in *b* is true. But if the case should be reversed and we should say,

b is included in *a*,

then it would be untrue, for not all Greeks are Athenians. Any statement that is either true or false is *meaningful;* *meaningless* if it be neither true nor false.

Suppose a set of objects, x, y, z, etc., and the relation of **implication**, and consider the statement,

x implies y.

Substitute here for x the expression, Socrates is a man, and for y the expression, Socrates is a mortal, and we have,

Socrates is a man implies
Socrates is a mortal,

the result, whether true or false, being at least meaningful. If we were to say, Socrates implies Greeks, we would say something that is neither true nor false. The expression would be meaningless because only of propositions (not of classes) can we say that the one *implies* the other.

Since the meaning of the objects and relationships with which a science deals will only appear when the science has been developed, it will not be possible to give an exact definition of the science with which we shall be concerned; but we may say that *logic* deals with propositions and with *inferences,* with classes and their *inclusion,* total or partial, with respect to one another.

PRELIMINARY DEFINITION OF THE SCIENCE

The most important matter with which logic is concerned is the matter of **inference.** Every man

reasons, and oftentimes, correctly enough, before he possesses any science of inference. It is the purpose of these chapters to enable the student to become aware of the *processes* which he employs habitually in his thinking; to enable him to distinguish, in as many cases as possible, a good argument from one that is fallacious; to enable him to argue with some consciousness of the misapprehensions that lead men to adopt erroneous opinions. *Inference* being, then, the most important matter with which the logician has to deal, we shall content ourselves with the following *preliminary* definition: *Logic is the science of inference necessary and probable.* Necessary inference corresponds broadly to what is called **deductive** logic, whereas probable inference is treated in another department of the subject, which is known as **inductive** logic. It is only with the science of necessary inference that the present work undertakes to deal.

LOGIC AND THE SCIENCES

Upon reflection the student will at once become aware that in all the sciences the drawing of inferences is customary and habitual, and we may adduce this fact as an additional reason for making a study of inference at first hand. Jevons remarks: "One name which has been given to Logic, namely

the Science of Sciences, very aptly describes the all-extensive power of logical principles. The cultivators of special branches of knowledge appear to have been fully aware of the allegiance they owe to the highest of the sciences, for they have usually given names implying this allegiance. The very name of logic occurs as part of nearly all the names recently adopted for the sciences, which are often vulgarly called the 'ologies,' but are really the 'logics,' the 'o' being only a connecting vowel or part of the previous word. Thus geology is logic applied to explain the formation of the earth's crust; biology is logic applied to the phenomena of life; psychology is logic applied to the nature of the mind. . . . Each science is thus distinctly confessed to be a special logic. The name of logic itself is derived from the common Greek word λόγος, which usually means *word*, or the sign and outward manifestation of any inward thought. But the same word was also used to denote the inward thought or reasoning of which words are the expression, and it is thus, probably, that later Greek writers on reasoning were led to call their science ἐπιστήμη λογική, or logical science; also τέχνη λογική, or logical art. The adjective λογική, being used alone, soon came to be the name of the science, just as Mathematic, Rhetoric, and other names ending in 'ic' were originally adjectives, but have been converted into substantives."

INTRODUCTION TO INFERENCE

INFERENCE AND ITS SIGNS

Whenever an inference is intended, we are usually made aware of the fact by the occurrence of some typical word: *hence; therefore; accordingly; if, then; implies; whence; it follows.* Thus:

> "If a strict definition of logic were stated at the outset, the student would not comprehend its intention,"

is an inference, by which it is intended to assert that the second part of the statement *follows* from the first. Or, again:

> "The definition of logic just given is exact. Hence the student must not expect to comprehend its full meaning at once."

But the presence of such words as these does not infallibly suggest that the first proposition *implies* the second. Suppose the case:

> "*If* there be any virtue, think on these things."—*Phil.* iv: 8.

This is in the form of an admonition or command, and, since it cannot be said to be either true or false, it is not properly a proposition at all.

It is important, in order to gain precise notions of the meaning of inference, to become aware that a proposition quite false in itself, may yet be truly implied by another. Thus it is untrue that Cæsar is an Athenian and untrue that Cæsar is a Greek, and yet the first proposition *implies* the second:

> If Cæsar is an Athenian,
> then Cæsar is a Greek.

The omission of the statement, "All the Athenians are Greeks," which is understood, if not expressed, a procedure common in ordinary speech, is called **enthymeme.** By this term we intend to signify that our speech is silent on a matter that is tacitly understood.

THE CASE OF FALSE PREMISES

Moreover, it is to be remarked that one or more assertions, untrue in themselves, may yet imply a proposition that is true. If we were to argue:

> Alexander is one of the heroes of the Iliad and all of these heroes died young; therefore, Alexander died in his youth,

our premises, or initial statements, would be false, although our conclusion would be true and the

argument would be formally correct. The fact to be observed in this connection is this: that the conclusion of an argument can only be asserted when the premises are true and the implication formally valid, *except* (as in the case above) when the conclusion is true in independence of the argument. We should then say (that is, in the case last contemplated) that the truth of the conclusion is based on *extralogical* information, that it is true in point of fact and not *because* of the premises.

We may, in this connection, call attention to an error into which uninstructed common sense sometimes falls, through a failure to recognize this distinction. A speaker, we will say, is trying to convince us of the wisdom of some political policy, or, it may be, the truth of some political theory. He begins with premises that we are altogether disinclined to allow; but, as the result of a series of cogent inferences, we find him, toward the end of his discourse, asserting matters that all of us agree to be so. Unconsciously, as we retrace his argument and discover each one of his steps to be validly taken and the outcome of his original statements to be true—unconsciously, I say, we are apt to conclude that there must be some truth in the premises with which he began. But such a conclusion would be in no way justified. The error involved in such a step is known as the fallacy of **asserting the consequent.**

NOTATION FOR CLASSES

We shall, in the chapters which follow, employ the small letters at the beginning of the alphabet, *a, b, c,* etc., to designate groups—Athenians, Greeks, Triangles, etc.—groups the members of which are ordinarily conceived by the aid of some quality or characteristic which they have in common. We shall thus learn to habituate ourselves to such expressions as,

All of the *a's* are *b's*

or, as we shall phrase it more commonly,

All *a* is *b,*

and by this we shall understand it to be asserted that all the members that belong to the *a* class are contained among the members of the *b* class. If *a* and *b* be particularized, so that the assertion becomes,

All metals are elements,

it will be true; whereas, if it were to read,

All metals are compounds,

it will manifestly be untrue.

In the light of this illustration it will be clear that the truth or untruth of assertions similar to this, like,

All *a* is *b*,
some *a* is *b*,
no *a* is *b*,
some *a* is not *b*,

will depend upon what specific meaning is assigned to the symbols *a* and *b*.

TRUTH-VALUES INDEPENDENT OF THE TERMS

But the student will have to habituate himself as well to expressions whose truth or untruth is *independent* of the meaning of *a* or *b*. Suppose that in the first and third expressions listed above, *b* should take on the specific meaning non-*a*, we should then have,

All *a* is non-*a*,
no *a* is non-*a*,

which would become, if we were further to particularize the meaning of *a*,

All Athenians are non-Athenians,
no Athenian is a non-Athenian.

It would be evident that the last of these expressions will be true for all possible values of *a* and

that the first will be untrue in the same way, without any restriction being placed on the meaning of *a*.

Similarly, the first member of our original set implies the second, that is,

> If all *a* is *b*,
> then some *a* is *b*,

quite in independence of what substantives may be substituted for these symbols. Thus,

> If all squares are circles,
> then some squares are circles,

and the implication holds even when *a* and *b* stand for **empty classes**—that is, for classes which contain no objects at all, for,

> If all square-circles are squares,
> then some square-circles are squares.

Another implication of a somewhat more general character than the cases that have just been cited, and whose truth is independent of the meaning of *a, b,* and *c,* would be,

> If some *b* is not *a*
> and all *b* is *c*,
> then some *c* is not *a*.

A case of this sort may occasion the student some difficulty when he meets it for the first time, but by continued attention to its sense he will soon be able to assure himself of its general truth. Particularized, it might read:

> If some scholars are not Englishmen
> and all scholars are cultivated,
> then some who are cultivated are not Englishmen.

By these examples we have sought to provide the student with a *preliminary* conception of the abstract nature of inference. To broaden and deepen this conception is a part of the task of the chapters which follow.

THE PROPERTIES OF A RELATIONSHIP

Relationships as well as their objects possess properties, and, since it is often important for the student of pure science to have these in mind, we shall enumerate those that are most characteristic. Incidentally, we shall illustrate further some of the distinctions already set down.

Corresponding to any relationship there will be a set of objects, of which it is meaningful to say that they stand in this relationship to one another. Such a set will be termed a **system.** A relation is said to be **reflexive** when it holds of any one of its

objects and that object itself. Thus, *implication* is reflexive, for,

If x (is true) then x (is true);

and so is *inclusion,* for,

(What is) a is included in (what is) a

and the same holds of numerical equality.

(The number) p equals (the number) p.

But *less than* is not reflexive:

(The number) p is less than p;

nor is *perpendicularity* reflexive, at least in the ordinary geometry:

(The line) m is perpendicular to m.

A relation is said to be **reciprocal** or **symmetrical** when, if it holds of any two of its objects, x and y, it holds also of y and x. Thus, *parallelism* and *perpendicularity* are symmetrical:

If x is parallel to y,
then y is parallel to x;
if x is perpendicular to y,
then y is perpendicular to x.

But, of events in time, *subsequent to* is not symmetrical:

> If p is subsequent to q,
> then q is subsequent to p;

nor is *to the right of* symmetrical:

> If a is to the right of b,
> then b is to the right of a.

A relation is said to be **transitive** if, when it holds of x and y and of y and z (three of its objects), it holds also of x and z. Accordingly, *implication* and *inclusion* are transitive, but *is the father of* is not transitive. Consider the assertion:

If p follows q and q follows r, then p follows r, p, q, and r being regarded as three events in time. Obviously, the statement will be verified if the events occur in the order,

$$r\ q\ p.$$

But suppose that they *actually* occur in the order,

$$p\ q\ r.$$

Then all of the *parts* of the original assertion, viz., p follows q, q follows r, and p follows r, are false. It is very important for the student to realize that the assertion *as a whole,* in spite of this fact, holds

true. The first two parts taken together form what is called the **antecedent** of the implication, and the last part is called its **consequent.** It may be remarked in general, that whenever one of the parts of the antecedent becomes false, or whenever the consequent becomes true, then the consequent *follows* in the particular case, whether it follows generally or not. There are six permutations of the three letters *p, q,* and *r.* The student will find it a valuable exercise to verify the *transitivity* of the relation *follow* (in point of time) by taking its objects in each one of their six possible orders.

MEANING OF DEFINITION

We have now made clear what it means for a relationship to possess or not to possess a given property. We remark, further, that it is by its properties that it is defined. Suppose three relationships are given and it is said of the first that it is symmetrical, but neither reflexive nor transitive; of the second that it is transitive, but neither reflexive or symmetrical; and of the third that it is reflexive and transitive, but not symmetrical.

Now, while you cannot say of the first relation precisely what it is—it might be the relation of *spouse* or the relation of *perpendicularity*—you may yet name a good many relations which it is not. For example, it is not *father of,* or *subsequent to,* or *to the right of.* But (and this is the important

thing) its possible meaning is delimited by the conditions imposed upon it. The second might be the relation *greater than* or it might be *subsequent to.* The third might be *implication* or it might be *inclusion.* In each case there will be a great many things which it could not be.

Our meaning will now be clear when we say that a relationship is defined when enough of its properties have been enumerated to distinguish it from whatever other relationships are in question, and that these properties are to be found by constructing all the true and all the false propositions into which this relationship may enter in a meaningful way. The task of any deductive science, then, is to completely develop its own system, for it is precisely within its own system that the propositions in question may be found. A deductive science, therefore, is defined by the task or problem which it sets for itself, and its *full* meaning, accordingly, will only appear when this task has reached completion.

Exercises

1. Are the following consequences justly drawn from the stated conditions:

   Upon experiment it is found that a musical note p and a musical note q cannot be distinguished by ear, and that the same holds of q and r. It is inferred that p and r are indistinguishable by ear.

   Two straight lines have no common perpendicular. It is inferred that they approach each other.

A candle is burned in air under a jar, until the oxygen is exhausted. It is inferred that the inert residuum ("nitrogen") is one chemical substance.

A chair and a lead ball of equal weight are held, one in each hand. It is inferred that they will be judged of equal weight.

The pitch of an engine whistle is rising. We infer that the train is approaching.

An ice vender has two weightless scales. He hangs one scale on the other and ten pounds of ice on the lower scale. We infer that each scale will register five pounds.

The number of prime numbers is less than the number of numbers, for not every number is prime.

A thick glass and a thin glass are filled with boiling water by different observers in different rooms. The observer with the thin glass infers that the glass in the other room did not break because his own did not.

2. Examine the following statements in order to determine their truth or untruth:

War is war and Germany is Germany.

"No man can lose what he never had."

A work of art is either moral or immoral.

"Hang sorrow! Care will kill a cat,
And therefore let's be merry."

"O wad some power the giftie gie us
To see oursels as ithers see us!"

It is true that not all prime numbers are odd.

If no proposition is true, then one proposition is true.

If the moon is green cheese, Cæsar and Socrates are the same person.

3. In the following propositions:

All metals are elements,
no elements are compounds,
some metals are red,
not all metals are white,

the relations are: "all, are," "no, are," "some, are," "not all, are." Classify these under the heads, reflexive, symmetrical, and transitive.

CHAPTER II

CLASSIFICATION OF TERMS

The word **term** is from the Latin *terminus* and is so designated because it forms one end of a proposition. For our purposes a term is synonymous with a **class,** a group of objects which have some characteristic in common, every substantive in the language being the symbol for such a group. If a term denotes a perfectly definite group of objects, it is called a **constant** term; if it stands indifferently for any class whatever, it is called **variable.**

THE PREDICABLES

Many are the ways in which terms may be classified, but only those divisions will be noticed which have some bearing on our subsequent theory and its applications. In the first instance, because of the importance of the terminology, we must describe the Aristotelian **predicables.** These are said to be the kinds of terms or attributes which may be predicated of any subject. They are called genus, species, difference, property, and accident. When a class is conceived as divided into a number of smaller classes it is called the **genus,** and each

smaller class that goes to make it up is called a **species** of the genus. Thus, if the genus be the class of numbers, odd numbers and even numbers are two species of the genus; if the genus be odd numbers, all prime numbers, except the number two, will be one of its species. It is important to observe that, while the genus contains more objects or individuals than the species, it is defined by fewer attributes. If the genus be number and the species be prime number, the genus is defined by whatever is characteristic of number, and the species by these same attributes with the addition of whatever is characteristic of a prime.

This double meaning of genus and species is fixed by different names. Thus, the individual things to which the term applies, comprise its meaning in **extension** (extent, breadth, or denotation). The qualities that serve to define the term comprise its meaning in **intension** (intent, depth, or connotation). Ordinarily, if we add qualities to a term—that is, if we increase its meaning in connotation or intension—we thereby diminish its meaning in denotation or extension. Two species of the genus substances would be elements and compounds, the first being gotten by adding the quality "chemically simple," or "not further reducible," to the genus. Again, two species of the genus element would be metals and non-metals. The first would be gotten by adding, it may be,

the characteristics, possessing luster and being an easy conductor of electricity. It is, therefore, usually said: As the connotation of a term is increased the denotation is decreased. The student must be warned, however, that this law only holds when the extension of the genus class is finite. Thus, if there be an infinite number of prime numbers, the denotation of "number" is not diminished by adding that the number is a "prime number."

The additional qualities required to distinguish the species from the genus are called the **difference,** and a class is supposed to be defined by the proximate genus (next higher genus) and the difference. The old manuals of logic, following tradition, speak often of a lowest species (*infima species*), a class not further divisible, and a highest genus (*summum genus, genus generalissimum*), which cannot in turn be taken as the species of a higher class. The modern counterparts of these are the **null class,** or class which contains no objects, such as square-circles or alien Americans, and the so-called **universe** of discourse, which contains all the objects that happen to be in question.

The remaining predicables, property and accident, are thus defined: A **property** is any quality that may be predicated of a class, and which, implicitly or explicitly, is essential to the meaning of the class. An **accident** is any quality which may be predicated of a class, but which is not essential

to the meaning of the class. It would be accidental for an isopod to be red, but essentially it must have equal legs. An odd number has the property of being not divisible by two; it is accidental to its meaning for it to be a prime. Water is essentially composed of hydrogen and oxygen and only by way of accident is it a solid, a liquid, or a gas.

REDUCTION OF TERMS TO THE CONCRETE GENERAL

Terms are, according to another important distinction, either singular or general. A **singular** term denotes an individual. Such names as London, Socrates, the Vatican, or the son of Napoleon I, Kant, or either one of the Kilkenny cats, designate a single object. A **general** term denotes any one of a number of objects. Cabbages and kings, monkeys and prime ministers would be examples of the case in question. In the logic of Peano, whose work has inspired much of the recent research in this subject, the relation of an individual to a class is conceived as different from the relation of a class to a class. What distinguishes the case of the inclusion of one class in another and the case of the inclusion of an individual in a class, is the property of transivity, which is taken to hold of the first relation, but not of the second. If I say, "Athenians are men and men are a class, therefore Athenians are a class," I say something that is not only meaningful, but true. But if I assert, "Soc-

rates is a man and men are a class, therefore Socrates is a class," I am supposed to say something meaningless, or at least something untrue, the conclusion being false and the premises being true.

We shall, however, in our subsequent theory reject this distinction of Peano's making, as one whose nature is extralogical, as one which turns on a question of fact or is a matter of application. That is, we shall say, "Socrates is a man" is exactly expressed by the phrase, "Every Socrates is a man," it being a matter of extralogical information—that is, a matter of fact—that there is only one Socrates. Or, to take a case in which a singular term occurs both as subject and as predicate, the proposition, "St. Paul's is the largest cathedral," is exactly, if awkwardly, expressed by, "Every St. Paul's is every largest cathedral," for it is a geographical fact that there is only one St. Paul's, and it is an arithmetical fact that there is only one largest member of a class. Accordingly, in translating grammatical expressions into the forms which are recognized by the logician, a singular term may always be reduced to a term which is general.

A further division of terms, which it is necessary to recognize, is the distinction between those which are abstract and those which are concrete. All general terms are either concrete or abstract. A **concrete** term is the name of a group of things conceived as individuals. An **abstract** term is the name

of a group of things conceived by the aid of a common property. It will not be necessary to multiply illustrations of this distinction. Truth and beauty are abstract terms; true things and beautiful things are concrete. If I say "Truth is beauty," the same meaning is advanced in the expression, "True things are beautiful things." Observe, then, that just as the singular term may always be reduced to the concrete general, so terms that are abstract may always be made concrete.

In concluding a classification of terms we must notice their division into those that are positive and those that are negative. All general terms are either positive or negative. A **positive** term connotes the possession of a quality. A **negative** term connotes the absence of a quality. The genus substances is made up of two species, elements and non-elements, and of these the first is positive and the second negative. But this distinction is relative, for elements and non-elements may be called, respectively, non-compounds and compounds. Whenever a negative term describes some class important in itself, language has generally invented a positive term to correspond. For the chemist non-elements are as important as elements, so that the word compounds has been invented to stand for that class. There are, however, notable failures to create such a word. The genus elements is divided into metals and non-metals, but there is no corresponding positive term

with which to designate this latter class. In general we observe that is an accident of language, whether or not a negative term possesses a synonyme which expresses its positive sense. Not all terms which have a negative prefix, however, convey a negative intent. Jevons remarks: "The participle *unloosed* certainly appears to be the negative of *loosed*; but the two words mean exactly the same thing, the prefix *un-* not being really the negative; *invaluable*, again, means not what is devoid of value, but what cannot be measured; and a *shameless* action can equally be called by the positive term, a *shameful* action."

UNIVERSE OF DISCOURSE

It is necessary to have very clearly in mind the distinction between a positive term and its corresponding negative, or the distinction between a class and its contrary (contradictory), if we are to understand the meaning of what is called the **universe of discourse,** a conception which now plays a very important rôle in the present-day logical theory. The view that in any argument there is presupposed a limited class, which stands for all of the objects under discussion, is due to De Morgan and we can not do better than to quote this author in full: "Let us take a pair of contrary names, as man and not-man. It is plain that between them they represent everything imaginable or real, in the universe. But

the contraries of common language usually embrace, not the whole universe, but some one general idea. Thus, of men, Briton and alien are contraries: every man must be one of the two; no man can be both. Not-Briton and alien are identical names, and so are not-alien and Briton. The same may be said of integer and fractions among numbers, peer and commoner among subjects of the realm, male and female among animals, and so on. In order to express this, let us say that the whole idea under consideration is the *universe* (meaning merely the whole of which we are considering parts) and let names which have nothing in common, but which between them contain the whole idea under consideration, be called contraries *in or with respect to that universe.* Thus, the universe being mankind, Briton and alien are contraries, as are soldier and civilian, male and female, etc.: the universe being animal, man and brute are contraries, etc."

Exercises

1. Give a connotative and give a denotative definition of prime numbers less than ten.
2. Name three species of the following genera, which together make up the whole: animal, plant, number, element.
3. Show how the law which connects the quantity of extension with the quantity of intension holds of the series: gold, metal, element, substance.
4. Are the following characteristics, that are predicated of the planets of our system, properties or accidents?
   There is a "regular progression of distances" of the planets from the sun (expressed as Bode's law). This breaks down at Neptune.

The plane of a planet's rotation practically coincides with that of the orbit of each—probably excepting Uranus.
The direction of rotation is the same as that of the orbital revolution—probably excepting Uranus and Neptune.

5. Does the term property, as used in the statement which follows, conform to the definition of this chapter?
The properties of the elements are periodic functions of the atomic weights;—elements arranged in a series of increasing atomic weights show steady increase or steady decrease in a property within any one period. Exceptions are iodine, tellurium, iron, cobalt, and nickel.
6. What universe of discourse is most naturally suggested by the following terms and their negatives? Odd numbers, straight lines, carbon compounds, protective tariffs, foreign policies.
7. Define the following words in terms of what you take to be the proximate genus and the difference: triangle, mammal, proposition, system, species.
8. Can the following words be termed *indefinables* in the above sense? Class, point, number, time, length.
9. Classify the following words under the heads, singular and general, abstract and concrete, positive and negative: unwieldy, Jupiter, protoplasm, Senate, incognito, respiration.

CHAPTER III

LOGICAL AND GRAMMATICAL FORMS

At the conclusion of our introductory chapter we were at pains to distinguish between statements whose truth or falsity depends upon the meaning of the terms, and those whose truth or untruth is independent of any meaning that our general terms, *a, b, c,* etc., may happen to take on. Statements of the first sort are commonly designated **propositional functions**; those of the second sort, **propositions.** In spite of this distinction we shall employ the word *proposition* to denote any sentence that is either true or false, and the word will now be understood to be defined in this more general sense.

Not all the sentences which the grammarian recognizes are propositions. Thus, the **interrogative,** the **hortatory,** and the **imperative** modes of expression will be sentences, but not propositions in the sense defined. It is neither true nor false to say,

> "What's Hecuba to him or he to Hecuba,
> That he should weep for her?"

Or to say,

> "Eat, drink and be merry."

Or, again,

"Stand not upon the order of your going,
But go at once."

Of any sentence, however, which the grammarian calls **declarative, optative, or exclamatory,** either truth or falsehood may be predicated. It is our purpose here to indicate the manner in which any proposition, of whatever grammatical form, may be expressed by means of the few simple forms which the logician recognizes. It is manifest, then, that our analysis need not concern itself with sentences of the interrogative, the hortatory, or the imperative type.

In the first place, we observe that any expression in the optative form may always be made declarative. If one were to say,

"Would that ignorance were bliss,"

he might, presumably, substitute for this, without changing the meaning in any way,

"I wish that ignorance were bliss."

Or, again,

"The devil damn thee black,
Thou cream-faced loon,"

is clearly susceptible to the same modification, it may be at the cost of some rhetorical advantage.

A reduction of the exclamatory to the declarative form is equally possible. Thus,

"A Daniel come to judgment!"

means, we may presume,

"Another Daniel's come to judgment,"

Or, again, the exclamation,

"How sharper than a serpent's tooth it is
To have a thankless child!"

may easily be made direct assertion. We may, therefore, confine our attention to declarative sentences alone.

In effecting a further reduction of grammatical forms, in order to show that the forms employed by the logician are sufficient for the expression of any truth, we observe that any verb other than the verb "to be" may be rendered by the copula, by absorbing its meaning into the predicate term. The expression "it rains" has always reference to some particular place as,

"It is raining in London,"

and this in turn is equivalent to

"London is a place where rain is falling."

If it be remarked that,

"A favorite has no friend,"

the same intention is expressed by,

"Every favorite is friendless."

The distinction of *number* offers, in turn, no real difficulty. The proposition,

"All the Athenians are Greeks,"

may be written in the equivalent form,

"Every Athenian is a Greek."

Moreover, the word "all" need not be replaced by "every," for we may say, at the cost of some awkwardness:

"All the members of the class Athenians is a member of the class Greeks."

Further, differences of *tense* are easily reduced to the *present* by attaching a date, or the suggestion of a date, to the predicate term. Thus, in place of,

"Achilles was celebrated as the swift of foot," it may be stated that,

"Achilles is celebrated in the 'Iliad' as the swift of foot."

The student will have no difficulty in effecting a reduction of other grammatical distinctions, such as those of *person, voice,* and *mood.* Assuming, then, that these reductions have been made, it will be taken for granted that any proposition, of whatever grammatical structure, may be cast into one or more of the relational functions that belong to the domain of which logic treats.

FORMS RECOGNIZED BY THE LOGICIAN

The logician recognizes the following propositions as necessary and sufficient for the expression of any truth:

(1) The **disjunctive** form (either, or),

either x or y,

(2) The **conjunctive** form (and),

x and y,

(3) The **hypothetical** form (if, then),

x implies y,

(4) The **categorical** forms (adjective of quantity copula),

A(*ab*)	All *a* is *b*,
E(*ab*)	No *a* is *b*,
I(*ab*)	Some *a* is *b*,
O(*ab*)	Some *a* is not *b*.

THE DOUBLE MEANING OF DISJUNCTION

The disjunction, *either, or,* in ordinary speech is ambiguous. If we were to say of such and such a person,

> "This man is either an Englishman or a Shakespearean scholar,"

we may not intend to exclude the possibility of his being both, educated Englishmen being supposed, we will say, to know the plays well, while the same assumption is not made of men of other nationalities. But if we were to say,

> "I know that I shall either like this man or dislike him very much,"

the intention is clearly to exclude the possibility of my liking him for some of his qualities and disliking him for others.

Here, however, the disjunction is not one between classes, as in our first illustration, but one between propositions, as in our second example. Commonly, when the options of every-day life are expressed in the form of a disjunction, when they are what William James calls options of the living, forced or momentous kind, we intend the sense: either the one or the other, *but not both.* Thus to transform some of James's illustrations,

> "This man is either Christian or he is agnostic," or
> "You must either accept this truth or go without it,"

the two parts of the disjunction *cannot both be true.*

If, on the other hand, I should meet a man whose knowledge of Shakespeare is extraordinary and should exclaim in surprise,

> "Either this man is an Englishman or else he is a Shakespearean specialist,"

I mean at least *one* of the parts of the disjunction to be true, and *possibly both.* The student should always bear in mind that this last case conveys the

true meaning of *logical* disjunction and that disjunction will always be understood in this sense.

Whenever the letters x, y, etc., are used to designate propositions, they are commonly taken to be true; that is, the words, *is true,* are understood to follow each one. Thus, if we assert,

either x or y,

we are only shortening the expression,

either x is true or y is true, or both,

and the shorter phrase,

x and y

means in expanded form,

x is true and y is true.

Whenever the proposition x or the proposition y is taken to be false ,the words, *is untrue,* will have to be expressly written down.

LAW CONNECTING CONJUNCTION AND DISJUNCTION

It will always be possible to express the denial of a disjunction—that is, the assertion that the whole statement, either x or y, is untrue, in the form of a conjunction of the two propositions involved. Thus, the untruth of

either x is true or y is true

is exactly rendered by

x is untrue and y is untrue.

This is an important fact and is one of the reasons for taking logical disjunction in the sense above defined. If we deny of some person "that he is either foreign born or that he is foreign bred," we assert the same thing when we say:

> "That he is foreign born is untrue and it is untrue that he is foreign bred."

In the same way the denial of the conjunction of two propositions may always be expressed as a logical disjunction. Thus the untruth of

x is true and y is true

is precisely equivalent to

either x is untrue or y is untrue.

If we deny "that this student is dull and that he needs no stimulus," we express the same thing by saying,

> Either this student is not dull or else he needs a stimulus.

This law which connects the conjunctive and the disjunctive forms may be expressed generally as follows: the denial of the disjunction of two propositions is the conjunction of the two separately

denied; the denial of the conjunction of two propositions is the disjunction of the two separately denied.

LAW CONNECTING DISJUNCTION AND IMPLICATION

There is another fact, which is universally assumed in modern logical investigations and to which it is right to direct the student's attention. This is a certain equivalence, which is assumed to exist between the hypothetical and the disjunctive forms. The assertions,

> "If he speaks not in jest, then
> he speaks in earnest," and
> "Either he speaks in jest or in earnest,"

are taken to convey precisely the same meaning. A general statement of this truth would be: x implies y is equivalent to the phrase, either x is untrue or y is true. In the subsequent chapters we shall express the *denial* of "x implies y" by means of the expression, "x does not imply y."

THE CATEGORICAL FORMS AND THEIR DIAGRAMMATICAL REPRESENTATION

It only remains to explain the categorical forms and the notation which has been introduced to represent them. In the propositions, A(ab), E(ab),

I(ab), and O(ab), the terms are the **subject** *a*, which is written first in the bracket, and the **predicate** *b*, which is written second, and the **term-order** is the order subject-predicate. When we wish to indicate that the term-order is not settled, we shall place a comma in the bracket between the terms. Thus, A(a, b) may mean either "all *a* is *b*" or "all *b* is *a*."

"All *a* is *b*" asserts that all the members of the *a* class are contained among the members of the *b* class, leaving it undetermined whether the members of the subject class are related to the members of the predicate class through identity, or exhaust only a part of the members of that class. Accordingly, the word *some,* in the sense of *some at least, possibly all,* is understood, if not expressed, before the predicate term, and it is this sense which the word will always convey in our subsequent theory. The meaning of the assertion, "all *a* is *b*," may be illustrated by the following diagram (Fig. 1), that is, if "all *a* is *b*" is a true proposition, then the class *a* is related to the class *b* in one of these two ways,

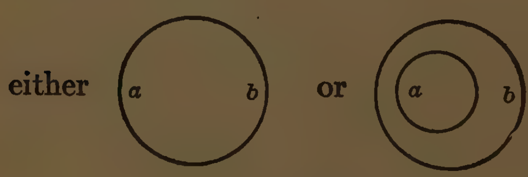


FIG. 1

The diagrammatic representation of the other categorical forms is given (in Figs. 2, 3, 4) below:

No *a* is *b*

Fig. 2

Some *a* is *b*

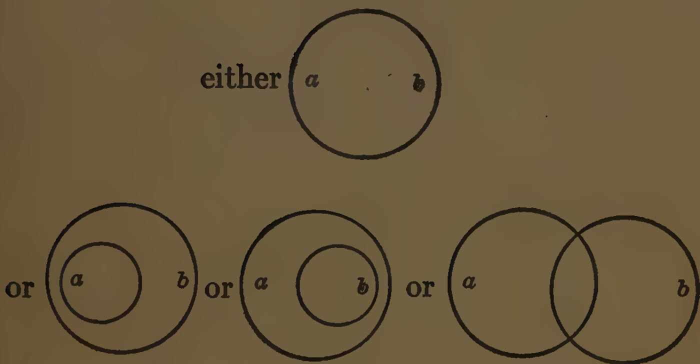


Fig. 3

Some *a* is not *b*

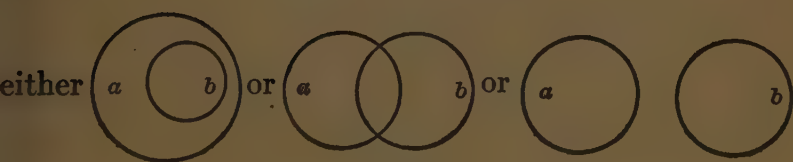


Fig. 4

These propositions are the ones employed in the classical logic, the science which has descended to us from Aristotle. The peculiar simplicity which is introduced into logic by the selection of this particular set of categorical forms depends upon the fact that the denial of any one involves the assertion of one of the others. Thus the denial of A is the assertion of O, and conversely; the denial of E is the assertion of I, and conversely.

Exercises

1. Are the following sentences propositions, and why?

> "Can it be possible that this old hermit has heard nothing of the report that God is dead!"
>
> —Nietzsche, *Thus spake Zarathustra.*

> "Necessity knows no law. . . . Behold the eleventh commandment, the message you bring to the world to-day, sons of Kant!"
>
> —Rolland, *Au-dessus de la mêlée.*

> "Render to Cæsar the things that are Cæsar's, and to God the things that are God's."
>
> —*Mark,* xii: 17.

2. Render the sense of the following sentence in the form of a conjunction:

> "The race is not to the swift, nor the battle to the strong."
>
> —*Ecclesiastes,* ix: 11.

and the sense of the following sentence in the form of a disjunction:

"Of making many books there is no end; and much study is a weariness of the flesh."

—*Ecclesiastes,* xii: 12.

3. Express the following disjunction in the hypothetical form:

"Either Bacon and Shakespeare are not the same person or else the moon is made of green cheese,"

and the following implication in the form of a disjunction:

"If you do not accept this truth, you must go without it."

4. Express the following sentences in categorical form:

"The heart hath its own reasons, which are unknown to reason."

—PASCAL.

"A nation free from prejudice soon becomes a free nation."

—CONDORCET.

"The brain in some sort digests impressions; it produces an organic secretion of thought."

—CABANIS.

5. The word *true* means true in all instances. Distinguish between the meaning of the phrase, *not necessarily true,* and the phrase, *necessarily not true,* and state general propositions which illustrate each meaning.

FALLACIES

CHAPTER IV

HISTORICAL CHANGES IN MEANING

Since ambiguity in the meaning of a word or phrase is perhaps the most fertile source of error, when we come to apply the rules of correct thinking, it is important for the student to be aware of some of the ways in which these ambiguities arise. **Semasiology,** whose problem it is to set forth the causes of these changes in meaning, is a field as yet but imperfectly explored. We shall be content to enumerate a few of the commonest cases in which old signs tend to disappear, and new ones arise to replace them.[1]

We note in the first instance that two *synonymes which are exact tend to differentiate in meaning.* De Quincey observes: "all languages tend to clear themselves of synonymes, as intellectual culture advances; the superfluous words being taken up and appropriated by new shades and combinations of thought evolved in the progress of society. And long before this appropriation is fixed and petrified, as it were, into the acknowledged vocabu-

[1] Some of the examples in this chapter are borrowed from Jevons, but by far the larger number are taken from Bréal, *Essai de Sémantique,* Paris, 1897.

lary of the language, an insensible *clinamen* (to borrow a Lucretian word) prepares the way for it. Thus, for instance, before Mr. Wordsworth had unveiled the great philosophic distinction between the powers of *fancy* and *imagination,* the two words had begun to diverge from each other, the first being used to express a faculty somewhat capricious and exempted from law, the other to express a faculty more self-determined.

When, therefore, it was at length perceived that under an apparent unity of meaning there lurked a real dualism, and for philosophic purposes it was necessary that this distinction should have its appropriate expression, this necessity was met half way by the *clinamen* which had already affected the popular usage of the words."

In Brittany, says the Abbé Rousselot, gardens were formerly called *courtils.* The rustic word which is now displaced as the usual term, is still used in a contemptuous sense. Similarly, with the introduction of the word *hotel,* the German *Wirtshaus, Wirtschaft,* has come to designate an inn of the less pretentious type. Again, to take an illustration from the philosophical dictionary, the Greek ἀρχαί (principles) and στοιχεῖα (elements) were certainly synonymous in the time of Thales, at least in the sense in which they were used by him. But this philosopher was severely reproached by Plato for not having distinguished between them.

LAW OF GENERALIZATION

Again, *words of special signification may take on a more general sense.* English *gain,* which has been influenced by the French *gagner* (to pasture), *le gagneur* (the cultivator), *le gain* (the harvest), means now, in either language, profit of any sort. Latin *pecunia,* originally wealth in live stock, came in the end to designate riches of whatever kind. French *temps* (time) meant at first heat (temperature), and afterward the weather, until finally the abstract idea of duration came to be attached to it. The Greek word χαραχτήρ denoted an engraving tool, but it soon came to be applied to the letters or signs engraved, a sense still preserved when we speak of the Greek characters. It was then extended by a sort of metaphor to whatever is regarded as the essential sign of any object whatever. The word *prince* has a sense much less special than the source from which it came, the *princeps senatus.*

Even proper names in certain instances may become generalized. Thus, in Rome *Cæsar* soon came to designate the emperor, and its modern derivative, czar, has even produced the abstract term, czarism. An adjective *Fabian* has been formed from the name of the Roman general Quintus Fabius Maximus, and any policy of procrastination or delay which postpones a decision is characterized as Fabian. The tower built on the island

of Pharos near Alexandria has produced in several languages the name "Pharos" as a synonyme for lighthouse. Again, while there was in the beginning but one sun and one moon, we now speak of the fixed stars as *suns* and we refer to the satellites of Jupiter as the *moons* of Jupiter.

LAW OF SPECIALIZATION

A tendency which is precisely the opposite of the one that has just been recorded is constantly taking place in the formation of a language. *Words of a general signification may take on a special or more restricted meaning.* Thus, in the Middle Ages the word *species* employed by the purveyors of drugs to designate the kinds of ingredients which they sold (saffron, clove, cinnamon, nutmeg) became, when it again entered the colloquial language, the French *épices,* English *spice.* German *Muth* (courage) originally of a more general signification, which is preserved in the derivatives *Hochmuth* (pride), *Grossmuth* (generosity) etc., and roughly rendered by English *mood,* probably derived its special meaning from words like *Rittersmuth, Mannesmuth,* etc. English *wit,* while tending to become specialized, still preserves its archaic sense of "shrewdness" or "intelligence" in the phrase *mother wit.* Many other examples might be given. *Urbs,* the name of Rome for the country folk of Latium, became, because of the Roman legions, the

name familiar to the whole of the ancient world. *Physician,* from φύσις (nature), has become so far specialized that a new word, *physicist,* had to be invented to express the original meaning. This word in turn has shown the same tendency, and there is now no commonly accepted term by which may be designated a scientist, whose interest is primarily in nature—*i.e.,* in nature as contrasted with man. The word *pope* (Latin *papa*) may have originated as a term of endearment (little father), employed, perhaps, in much the way in which the French now speak of Papa Joffre.

The following remarks of De Morgan deserve to be quoted in full: "The word *publication* has gradually changed its meaning, except in the courts of law. It stood for *communication to others,* without reference to the mode of communication or the number of recipients. Gradually, as printing became the easiest and most usual mode of publication, and consequently the one most frequently resorted to, the word acquired its modern meaning; if we say a man publishes his travels, we mean that he writes and prints a book descriptive of them. I suspect that many persons have come within the danger of the law, by not knowing that to write a letter which contains defamation, and to send it to another person to read, is *publishing a libel;* that is, by imagining that they were safe from the consequences of publishing, as long as they did not

print. In the same manner, the well-established rule that the first publisher of a discovery is to be held the discoverer, unless the contrary can be proved, is misunderstood by many, who put the word printer in the place of publisher. I could almost fancy that some persons think rules ought to travel in meaning, with the words in which they are expressed."

Sometimes an abstract word may become the name of an object. The Latin *vestis* (the act of clothing oneself) became in course of time the name of a particular garment. The Latin ending *-tas* (as in *dignitas, cupiditas*) served to form a substantive expressing a quality. But *civitas,* which meant at first the quality of being a citizen, came to designate the totality of citizens and finally stood for "the city" itself. German *Kind* came in course of time to mean "infant," though originally it referred to "the race."

TRANSFER OF MEANING BY ANALOGY

Besides the two processes described above as **generalization** and **specialization,** the senses in which a word may be employed may be merely *multiplied.* This transpires most commonly by the transfer of meaning through the use of **analogy** or **metaphor.** Thus, in Latin "intelligence" is like a point which penetrates (*acumen*), while "folly" resembles a blunt knife (*hebes*). At Rome there took place,

every five years, a census accompanied by a religious ceremony called "purification" (*lustrum, lustratio*). Since on this occasion the magistrates and the priests walked among the crowds, the verb *lustrare* came to mean "to pass in review." Cicero expressed astonishment that the Roman peasants should have given the name pearl (*gemma*) to the buds of trees. Actually the change by metaphor had been in the opposite sense, for pearls were so named because of their resemblance to buds about to burst. Again, *rivalis* designated neighbors on the opposite sides of a stream or who used the same water supply, but came to designate later on any sort of rivalry whatever. The word *influence* goes back to ancient astrological superstitions; it was supposed that a certain fluid escaped from the stars to predetermine the destiny of men and events.

Some of Archbishop Whately's examples of the transfer of meaning by analogy are worth recording. He says: "A blade, of grass, or of a sword, have the same name from direct resemblance between the things themselves. But instances of this kind are far less common than those in which the same name is applied to two things, not from their being *themselves* similar, but from their having *similar relations* to certain other things. And this is what is called analogy. Thus, the sweetness of a sound and of a taste can have no *resemblance:* but the word is applied to both, by *analogy,* be-

cause as a sweet taste *gratifies* the palate, so does a sweet sound, the ear. Thus, we speak in the analogical sense of the hands of a clock, the legs of a table, the foot of a mountain, the mouth of a river, . . . from the *similar relations* in which they stand to other things, respectively, in reference to use, position, action, etc.

"The words pertaining to *mind* may in general be traced up, as borrowed (which no doubt they all were, originally) by analogy, from those pertaining to *matter*: though in many cases the primary sense has become obsolete. Thus 'edify' in its primary sense of 'build up' is used, and the origin of it often forgotten; although the substantive 'edifice' remains in common use in a corresponding sense. When, however, we speak of 'weighing' the reasons on both sides—of 'seeing' or 'feeling' the force of an argument—'imprinting' anything on the memory, etc., we are aware of these words being used analogically."

Exercises[1]

1. How would you explain the fact that the first two words in the following lists are Saxon and the last two Norman?

   (a) home, hearth; palace, castle.
   (b) boor, churl; duke, count.
   (c) ox, deer; beef, venison.

2. How did *miscreant* (misbeliever) acquire its present sense?

[1] These examples have been taken from Trench, *On the Study of Words.*

3. How could *dunce* have been derived from Duns Scotus, the "subtle doctor," the great teacher of the Franciscan order?
4. As the result of a false science, *crystals* were so named because of their resemblance to ice, which was then supposed to have "lost its fluidity." Explain the words: jovial, saturnine, mercurial, disastrous, ill-starred.
5. According to an ancient theory of medicine, the disposition of mind and body depends upon a proper proportion of four principal moistures (humors). Explain: good humor, bad humor, dry humor.
6. Occasionally a name will embody some original error. Explain the words: America, Turkey, dinde (French), gypsies.
7. On being asked of what city he was, Diogenes replied that he was a cosmopolite. How must this reply have sounded to Greek ears?
8. By which one of the causes enumerated in this chapter would you explain the change in the following words from their primary to their secondary meaning: caprice, halcyon, voluble, temper, spirit, prude, loyalty, journal, minion, knave, roué?
9. What facts may be inferred from the history of the words: thrall, paper, stipulation, expend, calculate?

CHAPTER V

SEMI-LOGICAL AND MATERIAL FALLACIES

The term **fallacy** in the narrow or technical sense is used to designate some breach of the rules of correct inference; in the broader, more popular sense it denotes any one of the numberless ways in which men may fall into error. **Sophism, paradox,** and **paralogism** are inexact synonymes of this word and have a great variety of meaning. Thus paradoxical may be applied to an argument, which merely appears to be an offense against logic, or to a point of view which seems to offend common sense, or which only is beyond common apprehension. A sophism denotes ordinarily an argument which deceives not the author of it, but his opponent only, or which places the burden of the proof upon the latter. A paralogism may mean a special tendency of the mind to adopt an error of a particular kind, and in this sense it is employed by the German philosopher Kant. Fallacies commonly hinge upon an ambiguity in the meaning of terms or relationships. Thus it may appear *paradoxical* that this word and its synonymes should exhibit on their own part such a wide range of ambiguity.

PARADOX REMOVED BY EXTENDING THE MEANING OF TERMS OR RELATIONSHIPS

It is well known to the mathematician that he has often to extend the sense of his terms or of his symbols of relationship in order to take account of cases not suspected, when his definitions were set down. Thus, the primitive sense attaching to quantity will not permit of the interpretation of imaginaries. This, one of the prime difficulties which the beginner in algebra has to overcome, depends upon his instinctively holding to the *original* meaning of quantity, while striving to grasp the *new*. Many of the paralogisms of the older logicians depend upon their inability to generalize their conceptions, when they have met with special cases that resist interpretation. One would like, for example, to be able to say: If a is a class and b is a class, then what is a and b is a class, and what is either a or b is a class. But this will involve the conception of a class that has no members, and such a class has *peculiar* (*i.e., paradoxical*) properties. Thus the members of the class squares and the members of the class circles have no members in common and these latter are contained in and are also excluded from the members of any other class whatsoever. The corresponding notion of a proposition, that is true under no circumstances, is one with which the older logicians could not deal, and hence

for them paradoxical, although it is habitual and conscious in popular usage and is, therefore, recognized by common sense;

> "I will not be afraid of death and bane
> Till Birnam forest come to Dunsinane"

means: "I will not be afraid till what must forever remain untrue comes true; till an impossibility is possible; an emphatic way of saying, 'without qualification, I will not be afraid.'" Again:

> "Nay, had I power, I should
> Pour the sweet milk of concord into hell,
> Uproar the universal peace, confound
> All unity on earth,"

the meaning being, in part: if the consequent be false (I *shall* pour, etc.), then so is the antecedent false (I *have* the power), for an asserted implication is taken to be true. This sense is here rendered unambiguous by the use of the conditional.

EQUIVOCATION AND AMPHIBOLOGY

Many fallacies that arise because of the ambiguity of terms, and which are listed in the manuals of logic, are not intended seriously. Such are jests or puns. A famous case of punning is found in the conversation of Hamlet with the gravedigger:

Ham.—Whose grave's this, sirrah?
Clo.—Mine, sir.
Ham.—I think it be thine indeed, for thou liest in't.
Clo.—You lie out on't, sir, and therefore 'tis not yours: for my part, I do not lie in't, and yet it is mine.

Franklin's well-known remark (on the occasion, I imagine it was, of the signers attaching their names to the Declaration) might or might not be taken seriously, except for its logic, "If we do not hang together, gentlemen, we may expect, each one of us, to hang separately." There are cases in which one cannot be quite sure whether an ambiguity is intended or not. Suppose, for example, the assertion: "Shakespeare did not create the plays. They were conceived by another of the same name." If Mark Twain be the author of this saying, it is at once clear that an ambiguity is intended, but another author may intend to say that Bacon published them under an assumed name. A fallacy which depends upon the ambiguity of a single word is commonly called **equivocation**; if it consists in the ambiguity of a sentence or phrase, it is termed **amphibology**. As an illustration of the fallacy of equivocation consider the following argument for a protective tariff: "When we buy in a foreign country, we get the goods and they get the money, and

when we sell in a foreign country, they get the goods and we get the money. How much better, then, to buy and sell in our own country, for in that case we retain both the goods and the money." The following would constitute an equally good (*and* an equally bad) retort: "When we buy in our own country, the producer loses the goods and the consumer loses the money. But the consumers and the producers make up the entire community. Therefore, when we buy in our own country, we lose both the goods and the money."

An historical dispute as to whether logic is a science or an art depends, probably, upon an ambiguity in the original meaning of the word. Logic is from a Greek word, λογική, an adjective with some substantive understood, which in turn is derived from λόγος. This word possessed a twofold sense, denoting both man's thought and his expression of it, a distinction exactly rendered by the two Latin words, *ratio* and *oratio*. The same equivocation was carried by the derivative λογική, and hence arose, no doubt, the dispute as to whether logic deals with the laws of thought or with the laws of the expression of thought.

FALLACY OF ACCENT

Whenever the sense of an assertion is changed, because the emphasis is thrown on some particular word, the **fallacy of accent** occurs. A certain bar-

ber is supposed to have invited patronage by placing before his shop a sign which contained the following (unpunctuated) statement:

> "What do you think
> I'll shave you for nothing
> And give you a drink"

The visitor, once shaved and having demanded his drink, would be taken outside before the sign, which the barber would then read:

> "What! do you think
> I'll shave you for nothing
> And give you a drink?"

Another instance, which has even led to sectarian controversy, is the meaning of the phrase, "Drink ye all of it." Shall we say, "Drink ye *all* of it," or rather, "Drink ye all *of* it"?

FALLACY OF MANY QUESTIONS

To the fallacy of **many questions** are usually referred all cases in which too many meanings are contained, or in which the issue on that account is generally confused. A good example is the conversation in Hamlet between the grave-diggers. Here the first remark is not in the form of a question, but calls, none the less, for a reply. The fallacy might be termed equally well

the fallacy of **many statements.** The example will illustrate, too, what is called in logic a case of **non sequitur.**

> *First Clo.*—If the man go to this water and drown himself, it is, will he, nill he, he goes; mark you that; but if the water come to him and drown him, he drowns not himself: argal, he, that is not guilty of his own death, shortens not his own life.
> *Second Clo.*—Will you ha' the truth on't? If this had not been a gentlewoman, she should have been buried out of Christian burial.

Other fallacies are committed without the intention that they be taken seriously. Polonius conveys a sly hint to Hamlet when he says:

> "If you call me Jeptha, my lord,
> I have a daughter that I love passing well";

and Hamlet as slyly escapes by pretending that the remark contains a formal fallacy, for he rejoins:

"Nay, that follows not."

Nothing brings a conversation more abruptly to an end or more quickly disarms an opponent than the habit of taking him literally, for, arguing as it does a lack of imagination and even a lack of intel-

lect, he is at once aware that the discussion cannot be maintained on the projected level. Nor does this habit characterize the unlettered only. Many an excellent scholar will betray the essential poverty of his mind by traits which point the same moral, by his attachment to words rather than meanings, or by his scorn of a style that is elegant because elevated, or, again, let us say, by his liking for what he calls the *impersonal* (*i.e.* literal) narration of history. De Morgan remarks that "the genius of uncultivated nations leads them to place undue force in the verbal meaning of engagements and admissions, independently of the understanding with which they are made. Jacob kept the blessing which he obtained by a trick, though it was intended for Esau; Lycurgus seems to have fairly bound the Spartans to follow his laws till he returned, though he only intimated a short absence, and made it eternal."

FALLACY OF ACCIDENT

The same writer recounts the following tale from Boccaccio, a tale in which the man appears to have possessed more of *esprit* than his master, but who could hardly have expected to be taken too literally: "A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon table, the master desired to know what had

become of the other leg. The man answered that storks had never more than one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him next day into the fields where they saw storks, standing each on one leg, as storks do. The servant turned triumphantly to his master; on which the latter shouted, and the birds put down their other legs and flew away. 'Ah, sir,' said the servant, 'you did not shout to the stork at dinner yesterday; if you had done so, he would have shown his other leg, too.' " The servant was here guilty of what logicians call the **fallacia accidentis,** of predicating of roasted storks what can only be predicated generally of storks. But the fallacy of accident may easily involve us in serious difficulties. The illustration below, taken from De Morgan, deserves to be quoted in full:

"The law in criminal cases demands a degree of accuracy in the statement of the *secundum quid* which many people think is absurd. . . . Take two instances as follows: Some years ago, a man was tried for stealing a ham, and was acquitted upon the ground that what was proved against him was that he had stolen a portion of a ham. Very recently, a man was convicted of perjury, "in the year 1846," and an objection (which the judge thought of importance enough to reserve) was taken, on the ground that it ought to have been

"in the year *of our Lord* 1846." . . . In the two instances, which by many will be held equally absurd, a great difference will be seen by anyone who will imagine the two descriptions, in each case, to be put before two different persons. One is told that a man has stolen a ham; another that he has stolen a part of a ham. The first will think he has robbed a provision warehouse, and is a deliberate thief; the second may suppose that he has pilfered from a cook shop, possibly from hunger. As things stand, the two descriptions may suggest different amounts of criminality, and different motives. But put the second pair of descriptions in the same way. One person is told that a man perjured himself in the year 1846; and another, that he perjured himself in the year of our Lord 1846. As things stand, there is no imaginable difference; for there is only one era from which we reckon."

CONSCIOUS AMBIGUITIES

There is another large and important class of fallacies rather neglected, I think, by the logicians; arguments, which are not to be taken literally, but for a reason very different from the one that applies to the illustrations enumerated above. These are statements which are formally correct, but in which an ambiguity of terminology is *intended,* it may be for rhetorical purposes. Even the unlettered will not take you literally, if you remark that

"business is business." The formal correctness of the phrase tends to force its acceptance, but it is quite evident that more is meant than meets the ear. Again, if I assert that "man is a vertical animal," it will be clear that more than a mere tautology is meant. It is said of Lincoln, while making a tour of the trenches after a brisk fight, that he remarked, with evident disgust of the whole affair (I quote from memory), "Anyone who likes this sort of thing must enjoy it very much." If it be said that "a man is a man for all that," it is to call attention to the fact that a tautology is not always true; that rather "a man is not himself *sometimes.*" When the king and the others have left the play and Hamlet is left alone with Horatio, he says:

> "For if the king like not the comedy,
> Why then, belike, he likes it not, perdy,"

meaning that the burden of a bad conscience is the king's and not his.

Professor Stratton in an article appearing in the *Atlantic Monthly* says: "It is a prevailing belief that the mind is a convenient name for countless special operations or functions" and that these are independent. "When you have trained one of these you have trained that limited function and none other. What you do to the mind by way of education knows its place; it never spreads. *You train*

what you train." Here the formal correctness of the tautology seems to reinforce the argument. But this view of the character of mind ignores many important facts. "The psychological experiments which have so troubled the waters of education prove that normally *you train what you do not train.*" And, again, the conscious fallacy, the deliberate offense against logic, is correctly employed in favor of the opposite view.

In those verses of Lewis Carroll, which he calls "The Three Voices," the man in the piece, who has been accused by the lady of giving himself over to the exclusive instincts of his gourmandizing self, urges in his own defense that

> "Dinner is dinner, tea is tea."

His defense is undermined, however, by her resolve to take his statement only at its face value, for she replies:

> "Yet, wherefore cease, let thy
> scant knowledge find increase;
> say men are men, and geese are geese."

Here the intent is not only to overthrow the opponent's argument, to render his contention impotent by refusing his implied ambiguity; but also to make a joke at the erpense of logic itself, which is thus charged with giving us in its implications no information that we did not have before.

IGNORATIO ELENCHI

Again, an opponent may be disarmed for the moment by a statement that, while true, is irrelevant; what is called in logic the **ignoratio elenchi.** Thus Hamlet avoids telling his secret to Horatio and Marcellus by a reply that, to them at least, seems to have no bearing on the case. He says:

> "There's ne'er a villain dwelling in all Denmark
> But he's an arrant knave,"

and Horatio replies, quite properly:

> "There needs no ghost, my lord, come from the grave,
> To tell us this";

but Hamlet escapes again by simple agreement with this statement in its literal sense and by refusing to seek its implications. He rejoins:

> "Why, right; you are in the right;
> And so, without more circumstance at all,
> I hold it fit, that we shake hands, and part."

Agreement on the part of disputants is the end of discussion. It is said of the Frenchman Fontenelle, that he so far detested all forms of argument that he would refuse to differ with his opponent, em-

ploying habitually the phrase, *tout est possible,* whenever debate threatened. Another case in which an admission by an opponent may be taken unfairly, or, it may be, ironically, so that argument is effectually ended, is cited by De Morgan. "A writer disclaims attempting a certain task as above his powers, or doubts about deciding a proposition as beyond his knowledge. A self-sufficient opponent is very effective in assuring him that his diffidence is highly commendable, and fully justified in the circumstances."

AMBIGUITIES OF COMMON WORDS

Some of the ambiguities to which very common words are liable give rise to misunderstandings that are sometimes serious. Such words as the adjectives of quantity, *all* and *some,* the definite article, *the,* and the copula, *is,* come under this head. The assertions, "all of the angles of a triangle equal two right angles" and "all of the angles of a triangle are less than two right angles," are both true, if *all* is taken collectively in the first instance and distributively in the second. The phrases, "all of these twelve men are a jury" and "all of these men are liable to ~~be prosecuted~~" illustrate the same ambiguity. In Rousseau's conception of the social contract it is said that each member of society is called upon to surrender *all* his rights in order that the rights of *all* may be preserved—a result that may

appear paradoxical at first blush, until the equivocation in the use of the italicized word is noticed. In the phrase,

Chacun se donnant a tous ne se donne a personne,

the meaning is quite clear and unambiguous. The Latin *omnis* preserves the same double meaning. Thus the state of savage man is described as the *bellum omnium contra omnes.* The word, *both,* is similarly equivocal. If I say, "Both this man and his wife are either male or female," the case is true in the distributive but untrue in the collective sense of *both;* and the opposite will hold if I say, "This man can walk on both legs." But, "A man can hop on both legs" is true in either sense.

One of the chief difficulties of the logic of Hamilton depends on the ambiguity of the meaning of *some.* Of this word he says: "A remarkable uncertainty prevails in regard to the meaning of particularity and its signs. Here *some* may mean *some only—some, not all,"* and is "definite in so far as it excludes omnitude." Thus, "Some Greeks are Athenians." "On the other hand, *some* may mean *some at least—some, perhaps all."* Thus, "Some men are rational animals" where man is defined as a rational animal. If it be argued that "Socrates is a man and man is a class, therefore Socrates is a class," there are those who will find here an am-

biguity in the meaning of the copula or else in the meaning of the singular term. ither Socrates is regarded as a class of one member, they will say, or the relation of an individual to a class is to be distinguished from the relation of a class to a class. The definite article in English sometimes generalizes and sometimes individualizes. "The animal" is general when we speak of the "the animal in man," but otherwise when we say, "Have no fear, the animal will find his way home." Generally it is remarked, *"Man is unfaithful,"* but, *"The dog* is faithful to man." In Greek, in French, in German, on the contrary, the definite article is required before *man,* when the word is intended in the universal sense.

PETITIO PRINCIPII

It is generally true, and is, indeed, set down as one of the axioms of logic, that, if two propositions are true together, then either one of them may be assumed separately to be true. The statement will perhaps appear trivial, but a serious fallacy frequently arises in connection with it, and in the following way: Suppose that, in order to prove a given proposition, we should assume two others, such that one or both of the two assumed ones should be merely a disguised expression of the given one. If we should suppress the premises and assert the conclusion by itself, we should

then virtually assume the conclusion—that is, by assuming the right to suppress the premise equivalent to it. Such a fallacy is known as a **petitio principii.**

Serious instances of this fallacy are not uncommon in the history of science. Thus, most of the apparently successful efforts to demonstrate the so-called parallel axiom of Euclid are breaches of this rule. The demonstrator commonly takes for granted, intuitively, some principle which is equivalent to the result he seeks. The great geometer Gauss, writing on his efforts to effect this proof to his friend Wolfgang von Bolyai in the year 1799, says: "Certainly I have come upon much that for the majority would pass as a proof, but in my eyes demonstrates nothing." He then goes on to enunciate equivalent propositions that would be covertly taken for granted by many, but whose assumption would constitute a *petitio principii.* If the student will realize that for two thousand years mathematicians had struggled with this proof, the same fallacy being committed again and again, he will appreciate the seriousness of the difficulty. De Morgan relates that the mathematician Lagrange once wrote a memoir on the theory of parallels. While presenting it to the members of the French Academy, he withdrew the manuscript in the middle of the reading with the remark, *"Il faut que j'y songe encore."*

ACHILLES AND THE TORTOISE

In concluding this chapter on fallacies we shall include the case of a famous sophism which Zeno the Eleatic employed, in order to prove that motion is impossible. It is known as the paradox of Achilles and the tortoise. De Quincey gives the following account in one of his essays, and, as it cannot be better related, we shall quote him in full:

"Achilles, most of us know, is celebrated in the 'Iliad' as the swift-footed (ποδας ωκυς 'Αχιλλευς); and the tortoise, perhaps all of us know, is equally celebrated among naturalists as the slow-footed. In any race, therefore, between such parties, according to the equities of Newmarket and Doncaster, where artificial compensations as to the weight of riders are used to redress those natural advantages that would else be unfair, Achilles must grant to the tortoise the benefit of starting first. But if he does *that*, says the Greek sophist, then I, the sophist, back the tortoise to any amount, engaging that the goddess-born hero shall never come up with the poor reptile. Let us see. It matters little what exact amount of precedency is conceded to the tortoise; but say that he is allowed a start of one-tenth part of the whole course. Quite as little does it matter by what ratio of speed Achilles surpasses the tortoise; but suppose this ratio to be that of ten to one, then, if the racecourse be ten miles long, our

friend the slow-coach, being by the conditions entitled to one-tenth of the course for his starting allowance, will have finished one mile as a *solo* performer before Achilles is entitled to move. When the *duet* begins, the tortoise will be entering on the second mile precisely as Achilles enters on the first. But, because the Nob runs ten times as fast as the Snob, whilst Achilles is running his first mile, the tortoise accomplishes only the tenth part of the second mile. Not much, you say. Certainly not very much, but quite enough to keep the reptile in advance of the hero. This hero, being very little addicted to think small beer of himself, begins to fancy that it will cost him too trivial an effort to run ahead of his opponent. But don't let him shout before he is out of the wood. For, though he soon runs over that tenth of a mile which the tortoise has already finished, even this costs him a certain time, however brief. And during that time the tortoise will have finished a corresponding subsection of the course—*viz.,* the tenth part of a tenth part. This fraction is a hundredth part of the total distance. Trifle as that is, it constitutes a debt against Achilles, which debt *must* be paid. And whilst he *is* paying it, behold our dull friend in the shell has run the tenth part of a hundredth part, which amounts to a thousandth part. To the goddess-born what a flea bite is that! True, it is so; but still it lasts long enough to give the tortoise time

for keeping his distance, and for drawing another little bill upon Achilles for a ten-thousandth part. Always, in fact, alight upon what stage you will of the race, there is a little arrear to be settled between the parties and always *against* the hero. 'Vermin, in account with the divine and long-legged Pelides, Cr. by one billionth or one decillionth of the course,' much or little, what matters it, so long as the divine man cannot pay it off before another installment becomes due? And pay it off he never will, though the race should last for a thousand centuries."

It may be argued (as indeed it has been) that we can easily calculate the exact spot where Achilles will overtake the tortoise. But such a solution clearly misses the point. "Of course . . . it becomes easy, upon assuming a certain number of feet for the stride of Achilles, to mark the precise point at which that 'impiger' young gentleman will fly past his antagonist like a pistol shot, and being also '*iracundus, inexorabilis, acer,*' will endeavor to leave his blessing with the tortoise in the shape of a kick (though, according to a picturesque remark of Sidney Smith, it is as vain to caress a tortoise, or, on the other hand, to kick him, as it is to pat and fondle, or to tickle, the dome of St. Paul's)."

It is often said, somewhat patronizingly, that had Zeno grasped the modern notion of a differential coefficient, the limiting value of the ratio of two infinitesimals, there would have been no paradox.

This is the solution of Leibnitz and it is the one which De Quincey accepts. "The infinity of space in this race of subdivision is artfully run against a *finite* time; whereas, if the one infinite were pitted, as in reason it ought to be, against the other infinite, the endless divisibility of time against the endless divisibility of space, there would arise a reciprocal exhaustion and neutralization that would swallow up the astounding consequences, very much as the two Kilkenny cats ate up each other."

It must be remarked, however, that this solution is equally beside the point. The real difficulty, when the argument is properly stated, is to come to the end of an infinite series—that is, to come to the end of something that has no end by definition. The real fallacy, I believe, lies in an ambiguity in definition. The arguer defines an infinite series as one which has no *last term,* and later revokes the condition for his opponent, reinserting the last term as something that has to be passed through *for him.* Briefly the steps are these: "Achilles, in order to catch the tortoise, must pass through the last term of an infinite series. But an infinite series *has no* last term. Accordingly, in order to catch the tortoise, Achilles must do that which (by definition) he cannot do." The solution is to reject the major of this syllogism. If the series has no last term there is no need to pass through *a last term* in order to reach the limit. The last term, which is excluded

as a *possible* obstacle in the original definition, is reinvoked as a *real* obstacle for him against whom the argument is directed.

Exercises

1. Given an original illustration of each one of the fallacies specifically named in this chapter.
2. Examine the following statements and set forth clearly the sophism or paradox therein contained:

   A man accustomed to put his trust in dreams, one night dreamed that all dreams are vain. (From Jeremy Taylor's sermon on "The Deceitfulness of the Heart.")

   The Cretan Epimenides says that "all the Cretans are liars."

   The riches of a producer depend on the scarcity of his commodity. (From Bastiat, *Sophismes Économiques.*)

   Wealth consists in the abundance of things. The many who have little, combine to protect the few who have much.

3. Select from the following such as contain a "circle" in definition:

   Exceptive propositions affirm a predicate of all the subject with the exception of certain defined cases.

   An *affirmative* proposition is one in which an agreement is affirmed between the subject and the predicate.

   A *number* is anything which is the number of some class.

By the *mass* of a body is meant the quantity of matter contained in the body.

Force is that which tends to modify motion.

4. What ambiguities are implied in the following expressions:

"With respect to the appearance of this work (Fichte's *Characteristics of the Present Age*), I have nothing further to say to the Public than that I have nothing to say."

"Let anyone who reads this work without understanding it, assume no more than this: that he does not understand it."

"I have no other but a woman's reason,
I think him so, because I think him so."
—*Two Gentlemen of Verona,* Act I, Sc. ii.

"Non amo te, Sabidi, nec possum dicere quare;
Hoc tantum possum dicere, non amo te."
—MARTIAL, Ep. I., xxxiii.

5. Examine the argument given below in order to determine its validity or invalidity:

Differences cannot be expressed, for suppose two things, x and y to be different. Then we should say "x is not y. Now y is a way of existing and, consequently, not-y is a way of not existing; so that, in trying to express a difference, we have only said that x is a way of not existing."

IMMEDIATE INFERENCE

CHAPTER VI

THE UNIVERSE OF THE CATEGORICAL FORMS

We have seen that the categorical forms, A, E, I, and O, are composed of the terms (a and b), an adjective of quantity (*all, no, some, not all*) and the copula (*is*). In previous chapters we have noticed many cases in which the terms and their relations (*all is, no is, some is, not all is*) take on ambiguous meanings. In particular it has been said that the word *some* is to be taken in the sense of *some at least, possibly all.*

The student may well inquire by what right it is that we are allowed to understand this word in any sense we please. We reply that this meaning of the word is unambiguously forced upon us by the propositions which we say shall be true or untrue in our science. For example, we are going to say that

$$\mathrm{A}(ab) \text{ implies } \mathrm{I}(ab)$$

is a true, or valid, implication, and this would not be the case if *some* meant *some, not all.* We are going to say, too, that

$$\mathrm{I}(ab) \text{ implies } \mathrm{O}(ab)$$

is an untrue, or invalid, implication. But such an

implication would *follow,* would be true, if the word *some* were taken in this latter sense. We intend that the propositions which are valid or invalid in our system shall be confirmed as true or untrue by common sense. The meaning first given, *some, at least, possibly all,* is the interpretation which will be always verified in experience, when we come to apply our theory practically.

MEANING OF THE TRUE AND UNTRUE

There are two other words with which we shall have to deal, whose sense is best rendered ambiguous at the outset. Thus, *true* means *necessarily true,* true in all cases, true for all meanings of the terms. If the student were to represent the sense of "some a is b" by means of the diagram (Fig. 5) below

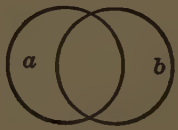


FIG. 5

it might then appear to him that the implication,

$$\text{I } (ab) \text{ implies O } (ab),$$

is true. That this is only an *apparent* truth, would

have been manifest at once, if he had employed instead either one of the diagrams of Fig. 6.

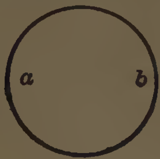
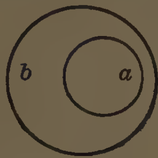
either  or 

FIG. 6

as his representation of "some *a* is *b*."

The word *untrue,* in turn means *not necessarily true,* not true in every instance, not true for all meanings of the terms; that is, there is at least one set of meanings of the terms which will invalidate the proposition in question. Thus, in order to become aware that the implication,

$$\mathrm{I}(ab) \text{ implies } \mathrm{O}(ab),$$

is not generally true, it would be enough to point to *either one* of the diagrams of Fig. 6, or to assign appropriate concrete meanings to the terms. Thus if *a* stands for metals and *b* stands for elements,

> "If some metals are elements,
> then some metals are not elements,"

the untruth of the general statement is at once manifest. If the illustration selected had been

> "If some red substances are elements,
> then some red substances are compounds,"

the untruth of the original implication would not have been shown.

THE PROPOSITIONAL UNIVERSE

These matters being clear, we may pass to the chief topic of this chapter, which is to explain what is meant by the *universe* of the categorical forms and to define certain technical expressions. In the last chapter, when we represented the categorical propositions by means of diagrams, we assumed that there is a certain analogy between the manner in which closed areas overlap and the manner in which classes overlap. This analogy was first pointed out by the mathematician Euler in a popular work on "natural philosophy" entitled *Letters Addressed to a German Princess,* and such diagrams are, accordingly, referred to as Euler's diagrams. If the student will examine the figures set down at the end of the last chapter, it will be intuitively clear to him, if he assumes this analogy, that any two classes, *a* and *b*, must be related in one, and cannot be related in more than one, of the following five ways (Fig. 7):

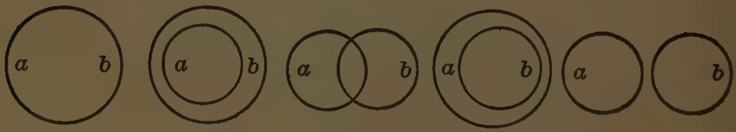


Fig. 7

These five cases may be conceived as five possibilities, one and only one of which can be realized in any particular case. That is, for any pair of concrete meanings of a and b (triangles and trilaterals, elements and compounds, Shakespearean scholars and Englishmen, etc.) one of the five possibilities is realized and the others remain unrealized. If I assert that one of these five representations is the true one, no matter what meanings the terms may take on, I assert something that is true. In the form of a disjunction the assertion would be, "Either the first, or the second, or the third, or the fourth, or the fifth possibility must be realized" for every meaning of a and b. This disjunction is called the **propositional universe,** or the **universe of the categorical forms,** or, again, the **logical sum** of all the possibilities.

If it is clear that at least one of these diagrams represents the true relation of a to b, it will be equally manifest that the relation of a to b cannot be represented in more than one of these five ways. If I assert the contrary to this last condition, I say something that is false. In the form of a conjunction this assertion would be, "Both the first (say) and the last (say) possibility are realized" at once. Such a conjunction is called the **propositional null,** or, again, the **logical product** of two or more of the possibilities.

CONTRADICTORIES, CONTRARIES, SUBCONTRARIES AND SUBALTERNS

By means of this conception of a propositional universe the more fundamental relations connecting the categorical forms may be established at once. Since "all *a* is *b*" asserts that one of the first two possibilities is realized and "some *a* is not *b*" asserts that one of the last three possibilities is realized, the disjunction,

"Either all *a* is *b* or some *a* is not *b*,"

—that is, the assertion that "either A is true or O is true," is precisely equivalent to the *propositional universe,* and is, therefore, of necessity a true statement. Similarly, the conjunction,

"All *a* is *b* and some *a* is not *b*,"

—is equivalent to the *propositional null,* and is false, for A and O contain no possibilities in common. Accordingly, A and O cannot both be true and cannot both be false.

In general, whenever two categorical propositions contain one or more possibilities in common, they may both be true, but not otherwise. Thus, I and O may both be true, but A and E cannot both be true. Whenever two categorical propositions, taken together, do not make up the universe of possibilities, they may both be false, but not otherwise.

Thus, A and E may both be false, but I and O cannot both be false.

We proceed to set down the following definitions, which will prove to be of great importance for our subsequent theory:

Two propositions which cannot both be true and cannot both be false are called **contradictories.**

Two propositions which cannot both be true but which may both be false are called **contraries.**

Two propositions which may both be true but which cannot both be false are called **subcontraries.**

Two propositions which may both be true and which may both be false are called **subalterns.**

The student who has made only a slight progress in his mathematical studies will still realize that we cannot classify the four assertions, A, E, I, and O, under these heads by a mere reference to a set of Euler's diagrams. The truths that we assume without demonstration must appear among our axioms, however "self-evident" they may otherwise seem. We accordingly assume:

Postulate 1.—A and O cannot both be true and cannot both be false.

Postulate 2.—E and I cannot both be true and cannot both be false.

From these assumptions it follows that A and O and that E and I are contradictory pairs. In the next chapter postulates and principles will be set down and theorems will be deduced from them,

by means of which it will be possible to say that A and E are contraries, that I and O are subcontraries, and that A and I, and E and O, are subalternate pairs. As a preliminary exercise, however, the student should scrupulously verify these facts for himself, employing the diagrams of Fig. 7 with this end in view. In order to facilitate this verification as well as to emphasize its importance, the figure is again reproduced on this page below.

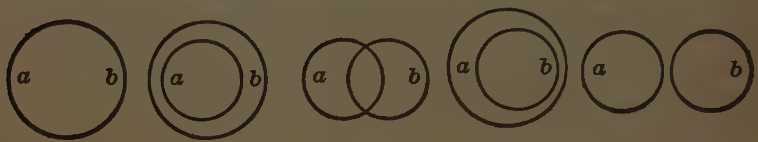


Fig. 8

EXERCISES

1. What propositions must be true and what ones must be false, when we take A, E, I, and O to be true in succession?
2. What propositions must be true and what ones must be false, when we take A, E, I, and O to be false in succession?
3. Draw a set of diagrams which will represent A, E, I, and O when the terms, *a* and *b*, have been reversed. In what cases does this alteration leave the original meaning unchanged?
4. In an argument your opponent has managed to establish the truth of a proposition, which is subcontrary to the one which you are maintaining yourself. How would you retort his contention?
5. Is it more desirable in an argument to establish the contrary of your opponent's view or the contradictory?
6. Assume that the first four of Euclid's axioms must be affirmed, but that either of the remaining two may be denied. If the fifth and sixth axioms are subcontraries, how many geometries different from Euclid's are possible?

CHAPTER VII

THE MOODS OF IMMEDIATE INFERENCE

In the hypothetical proposition, x implies y (if x is true, then y is true), the part x to the left of the word *implies* is called the **antecedent** and the part y to the right is called the **consequent.** Here x and y may stand for any sort of proposition, but if each one is a single categorical form, then we should replace x and y by a more definite notation, for example, "A(ab) implies I(ba)," "if E(ab) is true, then A(ab) is false," etc. Any implication of this latter specific type is known as **immediate inference.**

It will be recalled that the comma in the bracket between the terms is used in order to indicate that the term-order is not settled. Thus, just as O(a, b) may represent either O(ab) or O(ba), so all propositions like "A(a, b) implies A(b, a)" may have either one of two term arrangements. A difference between two forms of inference which is dependent on term-order alone is known as a difference of **figure.**

FIGURES OF IMMEDIATE INFERENCE

If the term-order in the antecedent is the same as the term-order in the consequent—that is, if,

for example, "A(a, b) implies A(a, b)" be written:

either "A(ab) implies A(ab),"
or "A(ba) implies A(ba),"

then "A(a, b) implies A(a, b)" is said to be expressed in the *first figure* of immediate inference. If the term-order in the antecedent is the reverse of the term-order in the consequent—that is, if, for example, "A(a, b) implies A(a, b)" be written,

either "A(ab) implies A(ba),"
or "A(ba) implies A(ab),"

then "A(a, b) implies A(a, b)" is said to be expressed in the *second figure* of immediate inference. It is clear that this implication will be true in the first, but untrue in the second figure, so that a difference of figure may very possibly involve a difference in the *truth-values* of the two cases.

ARRAY OF IMMEDIATE INFERENCE

It is evident that all the variants of immediate inference are to be gotten by permuting the four letters A, E, I, and O, two at a time, and by taking each letter once with itself. We should thus obtain sixteen distinct propositions of the type we are considering, as follows:

A (a, b) implies A (a, b)
A (a, b) implies E (a, b)
A (a, b) implies I (a, b)
A (a, b) implies O (a, b)
E (a, b) implies A (a, b)
E (a, b) implies E (a, b)
E (a, b) implies I (a, b)
E (a, b) implies O (a, b)

I (a, b) implies A (a, b)
I (a, b) implies E (a, b)
I (a, b) implies I (a, b)
I (a, b) implies O (a, b)
O (a, b) implies A (a, b)
O (a, b) implies E (a, b)
O (a, b) implies I (a, b)
O (a, b) implies O (a, b)

It will be convenient from time to time to leave unexpressed the word *implies* and the (a, b) and to write down the same set of sixteen implications in the following more abbreviated fashion:

AA	EA	IA	OA
AE	EE	IE	OE
AI	EI	II	OI
AO	EO	IO	OO

Each proposition of the set may be expressed in

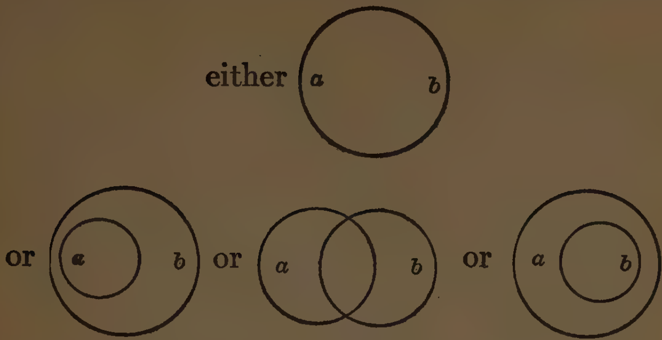
either the first or the second figure and there are, consequently, thirty-two possible forms of immediate inference. The entire set of thirty-two is said to constitute the **array** of immediate inference. Each member of the array is called a **mood** of the array. The true propositions of the array are called **valid moods** of the array. The remainder are called **invalid moods** of the array.

VALIDITY AND INVALIDITY DETERMINED BY EULER'S DIAGRAMS

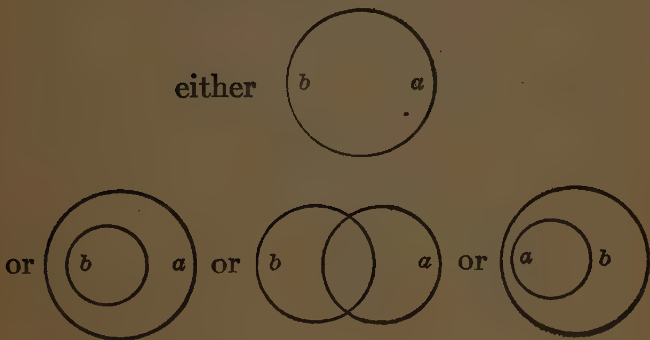
In order to determine precisely what are the valid and what are the invalid moods of this set, let us employ the method of inspection by means of diagrams which was explained in the last chapter. A few examples will suffice to illustrate this method. The student in completing the exercise which is here proposed, will do well to direct his attention to Fig. 7 of the last chapter.

(1) Consider the mood E(ab) implies O(ab), or, in our abbreviated notation, EO in the first figure. This asserts that if the fifth possibility (Fig. 7) is realized, then at least one of the last three possibilities is realized. It is intuitively evident then that EO in the first figure is a valid mood.

(2) Consider the mood I(ab) implies I(ba), or, in our abbreviated notation, II in the second figure. I(ab) is represented by



and I (ba) is represented by



These two modes of representation are identical, except that the diagrams do not appear in the same order. But, since the order in which the diagrams appear is irrelevant, it is intuitively clear that the mood is a valid one. If we had chosen to consider the mood AA in the second figure, it would have appeared at once that not all of the possibilities

contained in the antecedent are contained as well in the consequent, and the empirical invalidity of the mood would have been manifest at once.

In general, if the antecedent is of the *same form* as the consequent and the mood is valid in the second figure, then the antecedent (or, indifferently, the consequent) is said to be a **convertible** form. The operation of **simple conversion** consists in the interchange of subject and predicate and the proposition in question is said to be **simply convertible.** The student will discover that this operation is permissible in the case of E and I, but not in the case of A and O.

(3) Consider the mood, I(ab) implies A(ab). It is clear that if the terms are such as to realize either the third or the fourth possibility (Fig. 7), then I(ab) is true and A(ab) is false. Accordingly, the mood is invalid.

DEDUCTION OF THE VALID MOODS

This case leads us to make again the important observation: Since true means *necessarily true* and untrue means *not necessarily true,* it is enough to point to one diagrammatic representation of the antecedent which at the same time is not a diagrammatic representation of the consequent, in order to become aware of the invalidity of the mood. It is said of certain treatises of the Hindus on geometry, that the master, instead of offering a proof of the

separate theorems, was content, after stating the proposition, to draw the figure and write under it a word like *ecce*. The pupil was thus expected to gather intuitively the abstract or general truth from the observation of a single illustration. The student is well aware that the ideal of the Greek geometers was to deduce the theorems of the science from the fewest possible number of initial assumptions. Whether this ideal be a mistaken one or not, it has at least inspired the procedure of all science down to the present day. If we were to apply this historical contrast of the Greek and the Hindu geometers to ourselves, we might say that up to now our study of logic has been carried out on the Indian plan. Up to now we have been Hindu logicians, for we have been content merely to write *ecce* underneath our diagrams—a sort of Cartesian test, an application of the *clare et distincte percipio*. But from this moment forth we shall fashion our doctrine after the Helladian model. We shall deduce all the true and all the untrue variants of immediate inference by the aid of certain principles from the fewest possible number of initial postulates.

Definition.—Two propositions that cannot both be true and cannot both be false are said to be **contradictory.** By the postulates of the preceding chapter it follows that A(*ab*) and O(*ab*) and that E(*ab*) and I(*ab*) are contradictory pairs.

Principle I.—If in any valid mood antecedent and consequent be interchanged and each be replaced by its contradictory, a valid mood will result.

Postulate 1.—A (ab) implies A (ab),
Postulate 2.—A (ab) implies I (ab),
Postulate 3.—I (ab) implies I (ba),
Theorem 1.—O (ab) implies O (ab),
(from Postulate 1, by Principle I),
Theorem 2.—E (ab) implies O (ab),
(from Postulate 2, by Principle I),
Theorem 3.—E (ba) implies E (ab),
(from Postulate 3, by Principle I).

These are as many results as can be gotten from our assumptions. We therefore proceed with the introduction of an additional principle and with a definition which will make its application possible.

Definition.—If x implies y is a valid implication. then x is said to be a **strengthened** form of y and y is said to be a **weakened** form of x.

This definition of the meaning of strengthening and weakening is not to be taken in the traditional way, but in a more general sense. If it happens to be true, for example, that y implies x in addition to the fact that x implies y, then y will not only strengthen to x, but will also weaken to x. Thus, by postulate 3, I (ab) weakens to I (ba) and I (ba) strengthens to I (ab). But since postulate

3 can also be written I(ba) implies I(ab), this latter expression being only another way of writing II in the second figure, it follows that I(ba) is also a strengthened form of I(ab) and that I(ab) is also a weakened form I(ab). Again A(ab) weakens to I(ab) and I(ab) strengthens to A(ab) by postulate 2. In applying the principle about to be given, it must be noticed that the theorems just established give us the right to strengthen or to weaken in the same sense as do the postulates.

Principle II.—If in any valid mood the antecedent be strengthened or the consequent be weakened, a valid mood will result.

Theorem 4.—A(ab) implies I(ba),

(for Postulate 3 gives us the right to weaken the consequent of Postulate 2).

Theorem 5.—E(ba) implies O(ab),

(for Theorem 3 gives us the right to strengthen the antecedent of Theorem 2).

Theorem 6.—I(ab) implies I(ab),

(for Postulate 3 may be written in either of two ways, as explained above. Accordingly, Postulate 3 gives us the right to strengthen its own antecedent or to weaken its own consequent).

Theorem 7.—E(ab) implies E(ab),

(for, similarly, Theorem 3 gives us the right

to strengthen its own antecedent or to weaken its own consequent. Or we might have arrived at the same result by applying Principle I to Theorem 6).

DEDUCTION OF THE INVALID MOODS

We have, accordingly, by postulating the validity of three of the moods of immediate inference deduced the remaining seven by the aid of two principles. These two, as well as the two which are given below, will have to be assumed later on in any case, but in a more general form. The deduction of the invalid moods is felt as an exercise for the student. Since it will be necessary to postulate four of these moods as invalid, he will have eighteen theorems to deduce. The postulates and the principles of deduction are given below. It is only necessary to add that the additional results of theorems 4-7 (above) must be kept in mind when he comes to apply Principle IV (below).

Postulate 4.—A (*ab*) does not imply A (*ba*),
Postulate 5.—A (*ab*) does not imply O (*ba*),
Postulate 6.—A (*ab*) does not imply O (*ab*),
Postulate 7.—E (*ab*) does not imply I (*ab*).

Principle III.—If in any invalid mood antecedent and consequent be interchanged and each be re-

placed by its contradictory, an invalid mood will result.

Principle IV.—If in any invalid mood the antecedent be weakened or the consequent be strengthened, an invalid mood will result.

Theorems.—The other (18) invalid moods.

Exercises

1. The process by which we infer I from A or O from E in the second figure, is called *conversion by limitation* or *per accidens*. Cast the following into categorical form and convert by limitation:

   "A favorite has no friend."

2. When the terms of a proposition are simply converted, the resulting proposition is called the *converse* and the original proposition is called the *convertend*. What is the converse of the following:

   "No man e'er felt the halter draw,
   With good opinion of the law."

3. Immediate inference by *privative conception* consists in passing from an affirmative to a negative equivalent to it, or *vice versa*. Thus, "all metals are elements" is the same as "no metals are compounds"; "some elements are not metals" is the same as "some elements are non-metals." Effect this transformation in the statements of Exercises 1, 2, 4, and 5.
4. Conversion by *contraposition* consists in replacing the terms by their negatives and interchanging them. It is not permissible in the case of E and I. Convert by contraposition:

   "All that glisters is not gold."

5. Transform the following by privative conception and convert the result by contraposition and by limitation:

"No man can eat his cake and have it, too."

CHAPTER VIII

THE RULES OF IMMEDIATE INFERENCE

It is the custom of the traditional logic to formulate certain rules by whose aid the invalid moods of immediate inference may be detected immediately. These rules all turn upon the meaning of a *distributed* term. We begin, therefore, with an explanation of the sense of this conception, giving the definition at the outset and setting forth its application in the sequel.

DISTRIBUTED TERMS

Definition I.—Distributed terms are those modified, either implicitly or explicitly, by the quantitative adjectives "all" or "no." All others terms are **undistributed.**

In the first place, it is to be noticed that before the predicate of A and the predicate of I, the word "some" is unexpressed but understood. When we assert "all *a* is *b*," we mean: "all *a* is some (it may be all) *b*"; and the same remark holds of I. This fact may be more easily seen to hold of I, if we appeal to the property of simple convertibility of this form. When "some *a* is *b*" is written equivalently, "some *b* is *a*," the quantitative adjective

which is implicit before the predicate in the first case becomes explicit before the subject in the second case. The predicate of A and the predicate of I are, therefore, according to our definition, undistributed terms. It is equally clear that the subject of A is distributed, since it is modified by "all," and that the subject of I is undistributed, since it is modified by "some."

That the subject of E, "no *a* is *b*," is distributed, is at once apparent from our definition. But the distributed character of the predicate will be manifest as well, if we appeal to the property of the simple convertibility of E, established in the last chapter. Thus, "no *a* is *b*" being logically equivalent to "no *b* is *a*," the quantity of the predicate-term becomes explicit when it is made the subject. The same result would appear in another way if we were to assume the right to change E into A by privative conception, expressing it in the form "all *a* is non-*b*," and then in the form, "all *b* is non-*a*," the terms being then explicitly modified by the adjective "all." Accordingly, E distributes both its subject and its predicate by definition.

As regards the O-form, "some *a* is not *b*," it is apparent at once that the subject is an undistributed term, for it is modified by "some" and not by "all." But it is not so easy to see that the predicate is distributed. In order to become aware of this fact, imagine the part "some *a*" to represent

a fixed part of the *a* class. We may then imagine the contradictory of this part and designate it by the phrase, "non-some *a*." This part will constitute everything that is not "some *a*." The student will then be able, perhaps by the aid of a diagram which he may construct for himself, to understand that "some *a* is not *b*" is exactly rendered by the phrase, "all *b* is non-some *a*." The meaning which was implicit before the predicate in the first form has become explicit before the same term appearing as the subject in the new but equivalent expression. We conclude, then, that O distributes its predicate, but does not distribute its subject. These results appear in the following scheme, the distributed terms being printed in black letter:

A(**a** *b*)	E(**a** **b**)
I(*a* *b*)	O(*a* **b**)

We shall now state the first rule for the immediate detection of the invalid moods of immediate inference and we shall only introduce additional ones when it shall have been shown that this one is not in itself sufficient to effect our purpose.

Rule 1.—A form in which a given term appears undistributed does not imply a form in which that same term appears distributed.

Consider the mood A (*ab*) implies A (*ba*), or, in our abbreviated notation, AA in the second figure. The predicate of the antecedent is an undistributed term, but it appears distributed as the subject of the consequent. The mood is, consequently, declared invalid by the first rule. Again, in the mood AE in the first figure the subject term is distributed in both antecedent and consequent, but the predicate of the antecedent is distributed in the consequent. The mood is therefore declared invalid by the rule.

The student would now do well to construct the array of immediate inference for himself and to determine precisely just which moods in each figure come under the rule in question. He will find that some moods remain whose invalidity is not declared. We proceed, accordingly, to formulate two additional rules, which will prove exactly enough to effect our purpose. These will have to be preceded by definitions which will render them applicable.

AFFIRMATIVE FORMS AND NEGATIVE FORMS

Definition 2.—A form whose predicate is undistributed is called an **affirmative** form. By results already established it follows that A and I are affirmative. This definition will very possibly bewilder the student upon first consideration, for he will miss the motive which prompts it. He will rather have expected us to define an affirmative form

by means of a *synonyme,* after the fashion of the dictionary. Instead of that we have followed a procedure which is usual in science; we have selected a *property* which is characteristic of affirmative forms, but which does not characterize the others, and we have used this property in order to define them. Our next definition is:

Definition 3.—A form which distributes its predicate is called a **negative** form. By results already established it follows that E and O are negative forms. The two rules which remain to be stated are:

Rule 2.—An affirmative form does not imply a negative form.

Rule 3.—A negative form does not imply an affirmative form.

These three rules will be found *sufficient* for the purpose in hand, for it will be discovered that by means of them all the moods previously found to be invalid are declared untrue. That they are also *necessary*—that is, that no one of them can be dispensed with—will appear at once from the following consideration: Suppose that an invalid mood has been found that is declared invalid by the first rule and by no other rule. It is clear that this rule could not then be omitted from our list; and the same remark applies to the other two. The three rules are all necessary because we can point to at least one example which falls uniquely under each rule.

COROLLARY TO THE RULES

In addition to the rules there is a corollary which follows upon their assumption, and whose application depends upon the following:

Definition 4.—A form which distributes its subject is said to be **universal**. By results already established A and E are universal forms.

Definition 5.—A form which does not distribute its subject is said to be **particular.** By results already established I and O are particular forms. The facts that have now been made matters of definition are conveniently remembered by means of the mnemonic scheme which is given below, the distributed terms being printed in black letter.

	Affirmative	Negative
Universal	A (***a*** *b*)	E (***a*** ***b***)
Particular	I (*a* *b*)	O (*a* ***b***)

Corollary.—A particular form does not imply a universal form.

This theorem will be proven by showing that every one of the moods in question are declared invalid by one or more of the rules. Thus, OA in both figures is thrown out by the third rule and IE, IA and OE in both figures by the first rule.

A generalization which is based upon an examination of specific instances is said to be arrived at by **induction.** If the instances examined are all the instances that there are, as in the case of our three rules and the corollary, it is said to be **complete,** and the general truth arrived at is said to be based upon a **perfect** induction.

Exercises

1. Is the proof of the binomial theorem in ordinary algebra based upon a complete induction?
2. Does a generalization founded upon a single instance possess any degree of probability? Compare in this connection the following arguments:

   This box contains a dozen buttons; therefore, every box contains a dozen buttons.
   This solar system has eight planets; a fact that may well be true of every solar system.
   I am aware of the existence of my own mind by direct introspection; other men's behavior is apparently purposive and rational and like my own; therefore, other men, too, have minds.

3. Construct the array of immediate inference and place after each invalid mood the number of a rule or corollary that declares it to be invalid.
4. Prove that there is only one invalid mood which illustrates the second rule uniquely.
5. Make a list of examples which fall uniquely under the first and under the third rule.
6. Why is it that there is no unique illustration of the corollary?

SYLLOGISM

CHAPTER IX

MOODS AND FIGURES OF THE SYLLOGISM

We have now to study an array of a somewhat more general character than the one of immediate inference, and we may begin, not by describing it in the abstract, but by directing attention to a few specific examples. Consider the proposition:

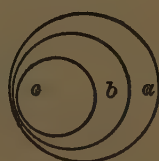
$$A(ba) \text{ and } A(cb) \text{ implies } A(ca),$$

and suppose that it is our desire to represent the antecedent as a whole. The diagrams below will evidently exhaust all the modes of representation that are possible.

either 

or 

or 

or 

It will be observed that each one of the four ways of representing the antecedent is at the same time a

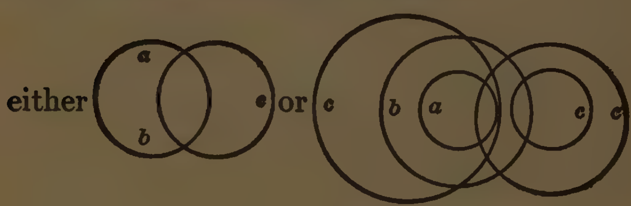
way of representing the consequent. Accordingly, if a, b and c are related as in the antecedent, then it follows that a and c are related as in the consequent, so that the original proposition is a valid implication.

RULE FOR CONSTRUCTING THE DIAGRAMS

The rule for constructing the diagrams which will represent the antecedent as a whole, is this: if the second form in the antecedent has (say) three modes of representation, then represent the first form completely three times (on three separate lines) and add to the first line the first way of representing the second form in the antecedent, to the second line the second way and to the third line the third way. The antecedent will then be completely represented as a whole. For example, consider the implication:

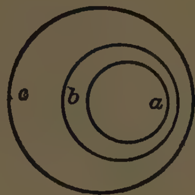
A(ab) and O(cb) implies O(ca).

Since O(cb) is represented in three ways (see Chap. VI, Fig. 7), we represent A(ab) completely three times. Now supply to the first line the first way of representing O(cb), that is,



and to the second line the second way of representing O (cb),

either 

or 

and, finally, to the third line the third way of representing O (cb),

either 

or 

It will be perceived at once that each separate manner of denoting *a*, *b* and *c*, as related in the antecedent, is also a manner of denoting *a* and *c* as related in the consequent. It is intuitively evident, then, as in the last illustration, that the implication is validly drawn. It will be necessary, perhaps, for the student to examine closely the more complicated diagram which appears in the first line, in order to satisfy himself that, together with the others, it exhausts all the possibilities there are.

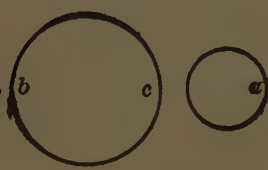
INVALIDITY DETERMINED BY DIAGRAM

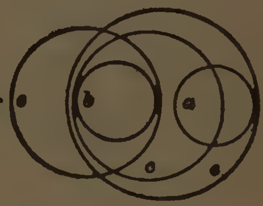
Since we have understood *untrue* to mean not necessarily true, it is clear that in order to perceive the invalidity of any proposition of the form under consideration it will be enough to point to a single representation of the antecedent which at the same time is not a representation of the consequent. When such a case is at hand, we are at once made aware of the implication's untruth.

Let us consider, then, a further case:

$$\text{E}(ab) \text{ and } \text{A}(bc) \text{ implies } \text{E}(ca).$$

The complete expression of the antecedent is:

either 

or 

But in the last diagram we have two separate instances of the untruth of $\text{E}(ca)$. Consequently, the implication is invalid.

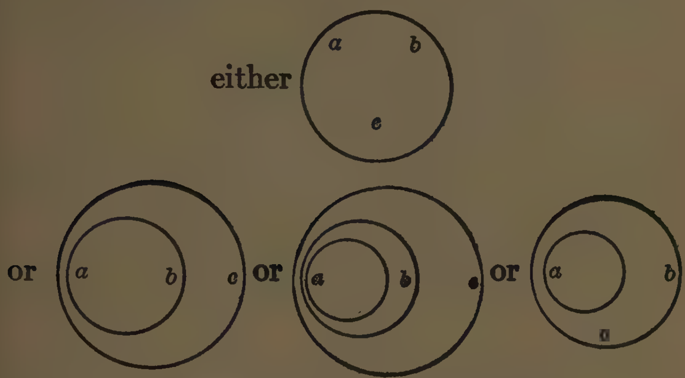
The first example which we examined above was,

$$\text{A}(ba) \text{ and } \text{A}(cb) \text{ implies } \text{A}(ca).$$

Consider now the following variation:

$$\text{A}(ab) \text{ and } \text{A}(bc) \text{ implies } \text{A}(ca),$$

and observe that the two cases differ in regard to their term-order. The complete representation of the antecedent is:



and three distinct instances will be observed in these diagrams of the untruth of the consequent, so that the implication is invalid. It is to be remarked, then, that the validity of an implication of the type under consideration depends not only upon the particular categorical forms which enter into it, but also upon the particular manner in which the terms are arranged.

DETERMINATION OF FIGURE

We shall now determine all the possible ways of arranging the terms. These will evidently be not more than eight in number, *viz.*,

ba	*ab*	*ba*	*ab*
cb	*cb*	*bc*	*bc*
ca	*ca*	*ca*	*ca*

ba	*ab*	*ba*	*ab*
cb	*cb*	*bc*	*bc*
ac	*ac*	*ac*	*ac*

It is clear that the two forms conjoined in the antecedent may be written in either order that we choose. In technical language this fact would be expressed by saying that the conjunctive relation of logic is commutative. We may then, if we wish, always write a specific one of the two first. *We agree as a matter of convention, always to write first the form which contains the predicate of the consequent.* Thus, we write

A(*ba*) and A(*cb*) implies A(*ca*),
rather than A(*cb*) and A(*ba*) implies A(*ca*).

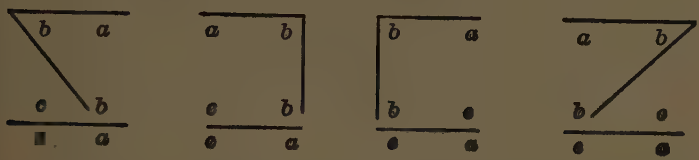
To accord with this convention, the second set above will have to be rearranged, thus:

cb	*cb*	*bc*	*bc*
ba	*ab*	*ba*	*ab*
ac	*ac*	*ac*	*ac*

We shall now show that the term-arrangements in this set are only a repetition of those in the first set above, but in a different order, so that it will turn out that there are only four distinct ways of arranging the terms.

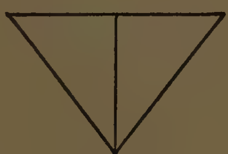
Suppose that we were to draw two lines, one connecting the terms in the categorical form written first in the antecedent and another connecting the

term which does not appear in the consequent. Then the first four varieties of term-order will appear thus:



and the second set will present the same varieties of figure, but with the first and last case reversed. The figures

will in each instance give a very clear geometrical image of the number of possible term-orders. The student will do well to commit to memory at once the four variations of the first set, which we shall constantly refer to as figures 1, 2, 3, and 4, respectively. The four figures are easily remembered as combined in an isosceles triangle standing on its vertex (see below).

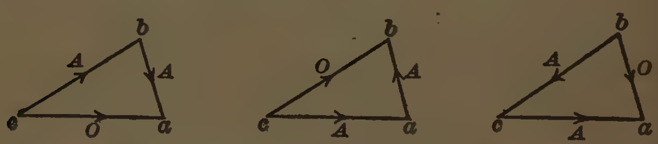


OTHER DIAGRAMMATIC METHODS OF DETERMINING FIGURE

While this method of determining the term-order will prove quite sufficient for all purposes, it is by no means the only device that might be constructed. Form a triangle with the term *a* at the end of the base to the right, the term *c* at the left, and the term *b* at the vertex above. Let an arrow indicate the direction of "flow" from subject to predicate, or, the order subject-predicate. Then if we choose

A(*ba*) and A(*cb*) implies O(*ca*),
A(*ab*) and O(*cb*) implies A(*ca*),
O(*ba*) and A(*bc*) implies A(*ca*),

these three propositions, whose invalidity the student may confirm for himself, would be represented by the diagrams given below:



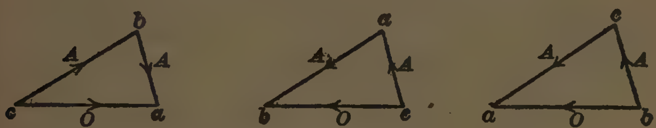
In the sequel the student will have to accustom himself to cases which do not at first appear to belong to any one of the conventional figures. Consider the three term-orders:

ba	*ab*	*ca*
cb	*ca*	*bc*
ca	*cb*	*ba*

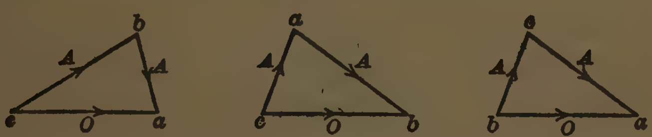
By following the original directions for determining the figure, it will be easy to recognize these as all variations of the first case. If we were to write these out, say in the form:

A(ba) and A(cb) implies O(ca),
A(ab) and A(ca) implies O(cb),
A(ca) and A(bc) implies O(ba),

these three equivalent statements would be represented by means of our triangles as follows:

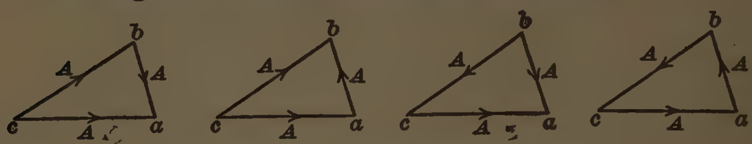


It will be noticed that in each instance the direction of "flow" as indicated by the arrows is continuous and in one direction from the subject of O to the predicate of O. The formal identity of the three cases will appear more clearly if the second and third figures be taken out of the plane of the page and turned over, thus:



Suppose, finally, that we should wish to represent by means of the triangles a single combination of

letters (AAA say) in each one of the four figures. Our diagrams would then appear as follows:



PARTS OF THE SYLLOGISM

We proceed now to summarize these results and to define a certain number of technical terms. If we imagine that the A's below may be replaced in any way by any one of the other letters, E, I, O, then the **syllogism** is a form of implication belonging to one of the following types:

1. A(ba) and A(cb) implies A(ca),
2. A(ab) and A(cb) implies A(ca),
3. A(ba) and A(bc) implies A(ca),
4. A(ab) and A(bc) implies A(ca).

These differences are known as the first, second, third, and fourth **figures** of the syllogism, respectively. The two forms conjoined in the antecedent are called the **premises** and the consequent is called the **conclusion.** The predicate of the conclusion is called the **major term** and points out the **major premise,** which by convention is written first in the antecedent. The subject of the conclusion is called the **minor term** and points out the **minor premise,** which by convention is written second in the antecedent. The term which is common to the premises

and which does not appear in the conclusion is called the **middle term.**

ARRAY OF THE SYLLOGISM

There will evidently be sixty-four syllogistic variations, obtained by taking the permutations of the four letters, A, E, I, O, three at a time. Each one of these may be expressed in each one of the four figures, so that we shall have two hundred and fifty-six cases in all to consider. These are known as the **moods** of the array of the syllogism. True propositions of the array are known as **valid moods** of the array. The remainder are known as **invalid moods** of the array.

In representing the array of the syllogism it will prove convenient, as in the case of immediate inference, to omit the word *and* and the word *implies,* as well as the parts (a, b), (b, c), (c, a) and to exhibit each mood as a simple combination of the three letters. In constructing the array, the best method to employ will be to add to each one of the sixteen permutations of the four letters, A, E, I, O, taken two at a time, each one of the four letters in succession. The array under each figure will then appear thus:

A A A	E A A	I A A	O A A
E	E	E	E
I	I	I	I
O	O	O	O

AEA	EEA	IEA	OEA
E	E	E	E
I	I	I	I
O	O	O	O
AIA	EIA	IIA	OIA
E	E	E	E
I	I	I	I
O	O	O	O
AOA	EOA	IOA	OOA
E	E	E	E
I	I	I	I
O	O	O	O

The student should now construct the array for himself and examine a great number of the moods in each one of the four figures, in order to determine the validity or invalidity of each one, by the method of inspection explained at the beginning of this chapter. We remark that six true propositions will be found under each figure, but in different positions in the array.

We shall assume now that the student has made a list of the valid moods of the syllogism, having applied the method of inspection to the two hundred and fifty-six possible cases. In order that he may verify his results, the six that are valid under each figure are set down below:

1.	2.	3.	4.
A A A	A E E	A A I	A A I
A A I	A E O	A I I	A E E
A I I	A O O	E A O	A E O
E A E	E A E	E I O	E A O
E A O	E A O	I A I	E I O
E I O	E I O	O A O	I A I

THE MNEMONIC LINES

The valid moods of the Aristotelian syllogism are conveniently remembered by means of the following mnemonic lines, the vowels in each separate word standing for the mood in question,

*Barbara, Celarent, Darii, Ferio*que prioris;
Cesare, Camestres, Festino, Baroko, secundæ;
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bokardo, Ferison, habet; quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo,
Fresison.

This mnemonic first appears in the *Summulæ Logicales* of Petrus Hispanus, who was afterward Pope John XXI, but without the line which records the fourth figure. He does not, however, profess to be the author of it. Several other versions are found in later writers. A Greek mnemonic of the same kind is inserted in early editions of the *Organon* of Aristotle. The moods not listed are

gotten by weakening the universal conclusions where they occur, to particular conclusions. Jevons remarks: "This device, however ingenious, is of a barbarous and wholly unscientific character; but a knowledge of its construction and use is still expected from the student of logic, and the verses are therefore given and explained." What Jevons overlooks is the fact, to be explained immediately, that this is really a first crude attempt at a deduction of the moods and, consequently, a matter of historical interest.

Besides the vowels in each word which stand for the mood in question, the initial consonants, as well as *m, p, s,* and *k,* have each a special significance. Thus, *s* means: convert simply in the preceding premise or conclusion. The letter *p* requires a conversion *per accidens* and *m* indicates an interchange of premises. By performing the indicated operations a mood in the second, third, or fourth figure will be reduced to the first, the initial consonant indicating to which mood of the first figure the reduction will be effected. Two illustrations will suffice to indicate the method.

(1) The mood *Cesare* is E A E in the second figure, or

$$\mathrm{E}(ab) \text{ and } \mathrm{A}(cb) \text{ implies } \mathrm{E}(ca).$$

Here only one change is indicated, that of simple

conversion in the major, the letter *s* following the first vowel. There results, evidently,

$$\mathrm{E}(ba) \text{ and } \mathrm{A}(cb) \text{ implies } \mathrm{E}(ca),$$

and this is the mood *Celarent* recorded in the first line.

(2) The mood *Bramantip* is A A I in the fourth figure, or

$$\mathrm{A}(ab) \text{ and } \mathrm{A}(bc) \text{ impress } \mathrm{I}(ca).$$

Converting the conclusion *per accidens,* which is required by the letter *p,* we get $\mathrm{A}(ac)$, and transposing the premises, as indicated by the letter *m,* the mood becomes

$$\mathrm{A}(bc) \text{ and } \mathrm{A}(ab) \text{ implies } \mathrm{A}(ac).$$

The student will have no difficulty, if he resorts to the method of triangular representation already explained, in recognizing this as A A A in the first figure, the mood recorded as *Barbara* in the first line of the mnemonic verse.

REDUCTIO AD IMPOSSIBILE

The moods *Baroko* and *Bokardo* require a special treatment known as indirect reduction, or *reductio ad impossibile,* and indicated by the letter *k.* In the old logic the premises were taken to be true by

presumption. Consider, now, the mood *Baroko* and suppose the conclusion does not follow from the premises. If O(ca) is false, then A(ca) is true. Combine this latter form with the major premise and we have

A(ab) and A(ca) implies A(cb),

a mood in *Barbara* whose conclusion contradicts the minor of *Baroko*. Our supposition, then, that the conclusion of *Baroko* does not follow, turns out to be impossible, and the proof depends upon the validity of *Barbara*.

INDIRECT REDUCTION BY OTHER PROCESSES

Indirect reduction may also be effected by other processes, either by attaching the negative particle to the predicate term, or else by the process of conversion by contraposition (see the exercises at the end of Chapter VII and the account of negative terms in Chapter II). Thus, converting the major of *Baroko* by contraposition and attaching the negative particle in minor and conclusion to the predicate term, we have

All *non-b* is *non-a*,
Some c is *non-b*,
Some c is *non-a*.

If *non-a* and *non-b* be represented by a' and b', respectively, this becomes

$$\mathrm{A}\,(b'\,a') \text{ and } \mathrm{I}\,(cb') \text{ implies } \mathrm{I}\,(ca')$$

which will be readily recognized as A I I in the first figure, a mood recorded in the first line of the mnemonic verses as *Darii.*

A similar process may be applied to *Bokardo* as follows: Attach the negative particle to the predicate terms and we should have:

$$\mathrm{I}\,(ba') \text{ and } \mathrm{A}\,(bc) \text{ implies } \mathrm{I}\,(ca').$$

This is the mood *Disamis,* which reduces to *Darii* in the usual way by following out the operations indicated by the letters.

Other moods besides *Baroko* and *Bokardo* are susceptible to this method of indirect reduction. Thus, *Camenes,*

$$\mathrm{A}\,(ab) \text{ and } \mathrm{E}\,(bc) \text{ implies } \mathrm{E}\,(ca),$$

reduces to *Barbara,* if we employ the two operations of conversion by contraposition and by negation—that is,

$$\mathrm{A}\,(b'\,a') \text{ and } \mathrm{A}\,(cb') \text{ implies } \mathrm{A}\,(ca'),$$

and the mood, *Ferison*—that is,

$$\mathrm{E}\,(ba) \text{ and } \mathrm{I}\,(bc) \text{ implies } \mathrm{O}\,(ca),$$

reduces to *Darii,* when we have converted simply in the minor premise—that is,

$$\mathrm{A}\,(ba') \text{ and } \mathrm{I}\,(cb) \text{ implies } \mathrm{I}\,(ca').$$

Exercises

1. Would any of the geometrical images employed in the determination of figure be changed for *Alice Through the Looking Glass?*
2. Reduce the moods of the other figures to those of the first, following the operations indicated in the mnemonic verses.
3. By indirect reduction change *Camestres* to *Barbara, Felapton* to *Darapti,* and *Fresison* to *Datisi.*
4. Reduce *Bokardo* by the method of *reductio ad impossibile.*
5. Remembering that contraries cannot both be true, reduce *Camestres* by the method of the last example.
6. If (*a b c*) is taken in cyclical order—that is, read *ab, bc, ca*—will any other figures result on permuting the three letters?
7. If *ab, bc, ca* stands for the fourth figure, what will be the effect of permuting these pairs in every possible way, regard being had to the proper order of the premises.

CHAPTER X

DEDUCTION OF THE SYLLOGISTIC MOODS

We shall now, as in the case of immediate inference, by postulating the truth of the smallest possible number of the valid moods of the syllogism. deduce the remainder by the aid of two principles. The assumptions which we shall have to make, are as follows:

Postulate 1.—A (ba) and A (cb) implies A (ca),
Postulate 2.—E (ba) and A (cb) implies E (ca),
Principle I.—If in any valid mood either premise and the conclusion be interchanged and each be replaced by its contradictory, another valid mood will result.

Principle II.—If in any valid mood a premise be strengthened or the conclusion be weakened, another valid mood will result.

Theorems.—The remaining (22) valid moods.

EXAMPLES OF DEDUCTION

When the student has carefully studied the examples which are set down below, he should be able to carry out the entire deduction without further aid and the work of doing this should have been

completed before reading the remainder of the chapter.

(1) Suppose that we were to combine the first postulate and the first principle. Interchanging minor premise and conclusion and replacing each by its contradictory we obtain

A(*ba*) and O(*ca*) implies O(*cb*).

Here the major term is *b*, so that the premises are in the normal order, the minor term is *c* and the middle term has become *a*. The figure is now determined in one of the ways already described, *viz.*,

$$\begin{array}{cc} b & a \\ c & a \\ c & b \end{array}$$

so that if our first theorem, AOO in the second figure, be written according to the original convention, our result becomes

Theorem 1.—A(*ab*) and O(*cb*) implies O(*ca*).

Similarly, by contradicting the major premise and the conclusion and replacing each by its contradictory, we should have obtained OAO in the third figure and this mood, if we employ the original convention of the third figure, becomes

Theorem 2.—O(*ba*) and A(*bc*) implies O(*ca*)

(2) The mood AOO in the second figure being now established as valid, we may apply to it either one of the principles in the same sense as to the

postulates. Let us begin by writing the mood with the terms ordered as in the original convention and, applying Principle II, let us strengthen the minor premise O (cb) to E (bc). This will be possible by applying a result of immediate inference already established, *viz.*,

E (bc) implies O (cb).

Accordingly, our next result becomes

Theorem 3.—A (ab) and E (bc) implies O (ca), or, AEO in the fourth figure is a valid mood.

(3) Suppose that we were to return now to the first principle and apply it to the result which has just been obtained. Contradicting major and conclusion and interchanging, we obtain immediately

A (ca) and E (bc) implies O (ab).

It is important that the student should not fail to observe that the premises are no longer in the normal order and that the normal order must be restored before the figure can be ascertained. Failure to make this change might result, as he will readily see, not only in a mistake in the figure but also in the mood. Our result is, accordingly, EAO in the fourth figure, or, if the terms be ordered as in the original convention,

Theorem 4.—E (ab) and A (bc) implies O (ca).

Had we chosen to contradict and interchange

minor and conclusion of AEO in the fourth figure, we should have obtained in the same way

Theorem 5.—A(ab) and A(bc) implies I(ca). Or A A I in the fourth figure is a valid mood.

It will now be observed that the application of Principle I to any mood in the fourth figure places the premises out of the normal order, but leaves the figure unchanged. Employing a more technical language, we should say that the fourth figure is invariant under Principle I.

Having deduced the twenty-two theorems, the student should set himself the exercise of deriving the valid moods under each figure separately, and he should try to arrive at each result by the fewest possible number of steps. In deducing those under the fourth figure, it will economize steps and so add to the elegance of his demonstration, if he will keep in mind the rule stated in the last paragraph. The following rules, which the student will do well to verify for himself, show the effect on mood and figure of contradicting and interchanging either premise and the conclusion.

Rules of Contradiction and Interchange

Contradicting and interchanging major and conclusion, we should have:

(1) The first figure changes to the third and conversely, and the premises remain in normal order;

(2) The second figure changes to the third with the normal order of the premises reversed;

(3) The fourth figure remains invariant with the normal order of the premises reversed.

Contradicting and interchanging minor and conclusion we should have:

(1) The first figure changes to the second and conversely, and the premises remain in normal order;

(2) The third figure changes to the second, with the normal order of the premises reversed;

(3) The fourth figure remains invariant, with the normal order of the premises reversed.

It will also be found advantageous to state in the form of rules the effect of simple conversion in either premise or in the conclusion.

Rules of Simple Conversion

(1) Simple conversion in the major premise changes the first figure to the second and conversely, the third figure to the fourth and conversely.

(2) Simple conversion in the minor premise changes the first figure to the third and conversely, the second figure to the fourth and conversely.

(3) Simple conversion in the conclusion changes the first figure to the fourth and conversely and the second and third figures remain invariant.

DEDUCTION OF THE INVALID MOODS

It remains in order to complete the solution of the syllogism, to deduce all of the two hundred and thirty-two invalid variants from the fewest possible number of initial assumptions. The most elegant way to proceed will be to begin with a single postulate and a single principle and to introduce further assumptions only when we are compelled to do so. We state, accordingly,

Postulate 3.—E (*ba*) and E (*cb*) does not imply I (*ca*),

Principle III.—If in any invalid mood a premise be weakened or the conclusion be strengthened, another invalid mood will result.

Let us begin by weakening the major to E (*ab*), since E (*ba*) implies E (*ab*); and secondly, by weakening the minor to E (*bc*). Finally, let us weaken the premises to E (*ab*) and E (*bc*) respectively. We shall then have established by postulate and theorem the invalidity of EEI in all four figures. If, now, the E- premises be weakened to O-premises and the I- conclusion be strengthened to an A- conclusion in every possible way, the untruth of

E E I	E O I	O E I	O O I
E E A	E O A	O E A	O O A

will have been established in each one of the four figures. The invalidity of thirty-one moods has, accordingly, been made to depend upon the invalid-

ity of E E I in the first figure alone. It should be noted in this connection that the application of Principle IV (below) to any mood in this set of thirty-two will yield no mood that is not already contained in the set and that Postulate 4 (below) will yield no mood of the set by either principle. We now introduce the second postulate and the second principle.

Postulate 4.—A(ab) and A(cb) does not imply I(ca).

Principle IV.—If in any invalid mood either premise and the conclusion be interchanged and each be replaced by its contradictory, another invalid mood will result.

The application of this principle will offer no difficulty that has not been already overcome, and no doubt the practice which the student has had in the derivation of the valid moods, will enable him to dispense with further illustrations here. Thus we should obtain at once the theorems:

a. A A A (second figure) by 4, iii,
b. A E O (first and third figs.) by 4, iv,
c. A E E (first and third figs.) by b, iii,
d. A I I (second and fourth figs.) by c, iv,
e. I A I (first and second figs.) by d, iii,
f. E A E (third and fourth figs.) by e, iv.

Other moods which follow from this postulate

and whose invalidity may be easily established in all four figures are:

EIE IEE IEO III IIA

and of this set of theorems it can be said that each one is independent of the original set of thirty-two.

In order to deduce the invalid moods that remain, it will be necessary to assume five other postulates, of whose independence the student will be able to satisfy himself by considerations similar to those set forth above. These are:

A A O (first fig.)
A A O (fourth fig.)
O A O (first fig.)
A A A (fourth fig.)
E E O (first fig.)

THE RULES OF SYLLOGISM

It was a part of the traditional treatment of the syllogism to formulate certain rules for the immediate detection of the invalid moods. We shall state these and we shall then prove that they are necessary and sufficient for the purpose which they effect.

Rule 1.—Two negative premises do not imply a conclusion.

Rule 2.—Two affirmative premises do not imply a negative conclusion.

Rule 3.—An affirmative premise and a negative premise do not imply an affirmative conclusion.

Rule 4.—Two premises in neither of which the middle term is distributed do not imply a conclusion.

Rule 5.—Two premises in which a given term occurs undistributed, do not imply a conclusion in which that same term occurs distributed.

These rules are *sufficient,* for they declare invalid all moods already recognized as invalid. They are all *necessary,* for we can point to at least one example that falls uniquely under each rule. In seeking unique illustrations of each rule, the student will do well to write out in full the mood to be examined and to underline the distributed terms.

Exercises

1. Beginning each time with *Barbara* and *Celarent,* deduce the valid moods of the second, third, and fourth figures.
2. From A A A (fourth fig.) deduce twenty-six other invalid moods of the syllogism.
3. Construct the array of the syllogism and place after each invalid mood the number of a rule that declares it to be invalid.
4. Make a list of examples that fall uniquely under each one of the five rules.
5. Prove that there can be only one invalid mood that will illustrate the second rule uniquely.
6. Show that, as the result of a complete induction of all of the moods in question, it follows from the rules as a corollary that two particular premises do not imply a conclusion.
7. Show similarly, that it follows from the rules that a universal premise and a particular premise do not imply a universal conclusion.

8. Prove that all moods of the first figure are invalid, wherein the minor is not affirmative or the major is not universal.
9. Prove that all moods of the second figure are invalid, wherein the major is not universal or both premises affirmative.
10. Prove that all moods of the third figure are invalid, wherein the minor is not affirmative or the conclusion is not particular.
11. Establish the following rules of the fourth figure:

    If the major is affirmative, the minor must be universal.
    If the minor is affirmative, the conclusion must be particular.
    Neither premise can be a particular negative nor can the conclusion be a universal affirmative.
    If one premise be negative, the major must be universal.

CHAPTER XI

THE HYPOTHETICAL SYLLOGISM

The forms of implication to which I shall now invite the attention of the reader are generally known as **conditional arguments.** They possess a peculiar attraction for minds whose interest is not readily aroused, save by the applications of a science, or by the enumeration of specific cases, for they conform to certain modes of rhetorical expression which seem natural, because they have become habitual. We cannot do better than begin with the citation of a few examples. The first we shall borrow from the logical compendium of Archbishop Whately. Suppose some one were to argue for the reality of miracles in the following way: "If no miracles had been displayed by the first preachers of the Gospel, they could not have obtained a hearing; but they did obtain a hearing; therefore, some miracles must have been displayed by them." He would, then, employ a form of argument known as the **hypothetical syllogism.** Or, again, suppose a certain theologian of opposite faith were to say: "If the doctrines of Calvin conform with the word of God, it is not necessary to publish them separately, and if they are at variance with the word of

God, they are wicked; but either they conform or they do not; accordingly, either these doctrines are wicked or their publication is unnecessary." An argument of such a form is known as a **dilemma.**

It would be well to examine these cases with especial care, regarding the form and the content of each and endeavoring to settle, without any of the apparatus of logic, the question as to whether the argument is good or bad. After the reading of this chapter it may be profitable to return to consider them again in the light of what will have been learned. The difference between our first judgments, together with the grounds on which we base them, and our later more sophisticated ones, which will then be founded on developed theory, will yield a rough measure of the practical value of this undertaking. The word *dilemma* is from the Greek *δι- two* and *λῆμμα assumption,* and the two parts of the disjunction appearing after the first semicolon are known as the *horns* of the dilemma. Some of the dilemmatic arguments which will be set down below are historically famous, and deservedly so, for it will not always be easy to put our finger on the "screw that is loose in our logical conundrum."

DILEMMA OF THE CROCODILE

The following account of the well-known dilemma denominated "The Crocodile" we shall quote in full from one of the essays of De Quincey:

"I recall at this moment a little metrical tale of Southey's, in which the *dramatis personæ* are pretty nearly the same, *viz.*, a crocodile, a woman and her son. In that case, however, the crocodile is introduced as a person of pattern morality, for the woman says of him—

" 'The king of the crocodiles never does wrong:
He has no tail so stiff and strong
Petitioners to sweep away,
But he has ears to hear what I say.'

Not so the crocodile known to the Greek dialecticians. *He* bore a very different character. If he has no tail to interfere with Magna Charta and the imprescriptible right of petitioning, he had, however, teeth of the most horrid description for crushing petition and petitioner into one indistinguishable pulp; and, in the particular case contemplated by the logicians, having made prisoner of a poor woman's son, he was by her charged with the same purpose in regard to her beloved cub as the Cyclops in the 'Odyssey' avows in regard to Ulysses, *viz.*, that he reserved him in his larder for an *extra bonne bouche* on a gala day. The crocodile, who, generally speaking, is the most uncandid of reptiles, would not altogether deny the soft impeachment, but, in order to sport an air of liberality which was far from his heart, he protested that, no matter for any private views which he might have dallied with

in respect to the young gentleman, he would abandon them all on one condition (but, observe, a condition which he privately held to be impossible for a woman to fulfil), *viz.,* that she should utter some proposition which was incontrovertibly true. The woman mused upon this; for though she knew of propositions that no neutral party could dispute—as this, for instance, that crocodiles are the most odious of vermin—it was evident that her antagonist would repel *that* as an illiberal and one-sided personality. After some consideration, therefore, she replied thus—'You will eat my son.' There and then arose in the crocodile's brain a furious self-conflict, from which it is contended that no amount of Athenian chicanery would ever deliver him; since, if he *did* eat her son, then the woman had uttered a plain truth, which the crocodile himself could not have the face to deny, in which case (the case of speaking truth), he had pledged his royal word *not* to eat him; and thus he had acted in a way to make the word of a crocodile, or his bond, or even the tears of a crocodile, a mere jest among philosophers. On the other hand, if in contemplation of these horrid consequences he did *not* eat her son, then the woman had uttered a falsehood in asserting that he would, and it became a royal duty in *him,* as a guardian of morality, to exact the penalty of her wickedness. . . . Truth absolute was provided for; in that case the son was to be spared.

Absolute falsehood was also provided for; in that case the son was to die. But truth conditional was *not* provided for. Supposing the woman to say something contingent on a case that might or might not be realized, then it became necessary to wait for the event. But here there was no use in waiting, since, whichever of the two possible events should occur, either equally and irretrievably landed the crocodile in a violation of his royal promise."

CONSTRUCTIVE HYPOTHETICAL SYLLOGISM

The **constructive hypothetical syllogism** is of the following general form:

> If x is true, then y is true;
> but x is true;
> therefore y is true.

This argument is called *modus ponens,* or the mood which affirms. Here the minor asserts the antecedent of the major; or, otherwise, what is asserted only hypothetically in the major is asserted absolutely or without qualification in the minor. This condition, of course, allows us to *suppress* the antecedent altogether and to assert the consequent by itself. The general rule is: whenever the antecedent is verified, the consequent is verified itself, provided the implication is a valid one. Thus suppose that I say: if the spring is late, the fruit crop will be abundant; but the spring *is* late; therefore, the

fruit crop will be abundant. The conclusion may be asserted by itself, if the premises are granted.

But it is important to observe that the conclusion *follows* whether the premises are true or not. Let us suppose the possible cases:

(1) x true and y true;
(2) x true and y false;
(3) x false and y true;
(4) x false and y false.

In the first case the major and minor are true and the conclusion may be asserted without qualification; in the second case the major is false and the minor is true and the conclusion cannot be asserted by itself; in the third case the major is true and the minor is false and the conclusion cannot be asserted by itself; in the fourth case the major is true and the minor is false and, again, the conclusion cannot be asserted by itself. Observe, however, that in the third case the conclusion, while it cannot be asserted because of the argument, because the premises cannot be suppressed, may yet be asserted on *extra-logical* grounds—*i.e.,* because it is true in point of fact; and the same remark applies to the first case.

PARADOX OF TRISTRAM SHANDY

In order to illustrate the "common-sense" tendency to suppress the antecedent when the implication seems to be a valid one, we shall quote an argu-

ment of Mr. Bertrand Russell's, an argument known as the paradox of Tristram Shandy: Mr. Russell says: "Tristram Shandy, as we know, employed two years in chronicling the first two days of his life, and lamented that, at this rate, material would accumulate faster than he could deal with it, so that, as years went by, he would be farther and farther from the end of his history. Now I maintain that, if he had lived forever, and had not wearied of his task, then, even if his life had continued as eventfully as it began, no part of his biography would have remained unwritten. For consider: the hundredth day will be described in the hundredth year, the thousandth in the thousandth year, and so on. Whatever day we may choose as so far on that he cannot hope to reach it, that day will be described in the corresponding year. Thus any day that may be mentioned will be written up sooner or later, and therefore no part of the biography will remain permanently unwritten. This paradoxical but perfectly true proposition depends upon the fact that the number of days in all time is no greater than the number of years."

Leaving out of account the question as to whether or not there is a paradox involved in the comparison of the two infinites, let us examine the following hypothetical syllogism: "If Tristram Shandy should live forever, and should not weary of his task, he will complete his biography; but it is agreed

that he shall live forever and shall not weary of his task; accordingly, he will complete his biography." The argument being formally correct, the only question is whether or not the premises may be suppressed and the conclusion asserted by itself. But in the minor it seems that we have agreed that an impossibility shall transpire; not because one cannot live forever for biological reasons, but because one cannot pass through each term of an infinite series, because one cannot come to the end of something that has no end by definition. We have agreed in the minor to regard an impossibility as possible, to regard a false proposition as true. There is no paradox, as soon as we have seen that the premises cannot be suppressed and that, therefore, the conclusion cannot be asserted by itself.

FALLACY OF AFFIRMING THE CONSEQUENT

A familiar fallacy, known as the **fallacy of affirming the consequent,** may be conveniently cited in this connection. Thus, suppose one were to argue:

"If the study of logic furnishes the mind with a multitude of useful facts it will deserve cultivation; now, we agree that it deserves cultivation; accordingly, it must furnish the mind with a multitude of useful facts."

This argument would be formally fallacious. The premises might easily be regarded as true and the conclusion as false under the same circum-

stances, for the consequent might follow from any one of a great number of antecedents. The study of logic might deserve cultivation as an aid to forensics or to legal studies; or, because its history is connected with the general history of philosophical speculation, or, because its technical vocabulary has passed into common use and cannot be properly understood by the mere help of a dictionary; or, because its foundations are presupposed by those of arithmetic and geometry; or, because constituting one of the few intellectual disciplines of the Middle Ages, a knowledge of it casts a flood of light upon the workings of the mediæval mind; and so on for any number of other reasons that might be enumerated.

A similar fallacy, and one not mentioned, so far as my knowledge extends, in the manuals of logic, consists in the inference that, because the conclusion of a true syllogism may be asserted, at least one of the premises may be asserted to be true. This we shall call the **fallacy of affirming the conclusion.** Consider the following argument:

"If all the Troglodytes are virtuous and this man is a Troglodyte, then this man is virtuous; but this man is virtuous; therefore, this man is a Troglodyte, or else all of the Troglodytes are virtuous."

The Troglodytes of the *Lettres Persanes* of Montesquieu were a mythical people, for the most part so savage and evil that the notions of human

justice and equity had ceased to operate practically in their relations with one another. They perished of their own injustice, with the exception of two families, who "were humane and loved virtue." The argument is evidently fallacious, but part of the antecedent of the major, which is a true syllogism, is false.

DESTRUCTIVE HYPOTHETICAL SYLLOGISM

The **destructive hypothetical syllogism** is of the following general form:

> If x is true, then y is true;
> but y is untrue;
> therefore x is untrue.

This argument is called *modus tollens,* or the mood which denies. Thus: "If perfect justice prevailed on earth, then virtue would receive its reward in this life; but virtue is not compensated for here below; therefore, perfect justice does not prevail on earth."

Voltaire, ridiculing the dictum of Leibniz that "all things are for the best in the best of all possible worlds," prefers to believe in a finite rather than a wicked God. An argument commonly given is: "If God were both omnipotent and good he would moderate the grosser evils in the world; but he does not do this; accordingly, either he is an evil being, or else his power is limited."

FALLACY OF DENIAL OF THE ANTECEDENT

The characteristic fallacy to be noticed in connection with this type of argument is the **fallacy of denial of the antecedent.** We quote from Jevons:

" 'If the study of logic furnished the mind with a multitude of useful facts like the study of other sciences, it would deserve cultivation; but it does not furnish the mind with a multitude of useful facts; therefore it does not deserve cultivation.' This is evidently a fallacious argument because the acquiring of a multitude of useful facts is not the only ground on which the study of a science can be recommended. To correct and exercise the powers of judgment and reasoning is the object for which logic deserves to be cultivated, and the existence of such other purpose is ignored in the above fallacious argument."

A valid argument, of a form more general than the destructive hypothetical syllogism, may be described as follows:

> If x and y are true, then z is true;
> but x is true and z is false;
> therefore, y is false.

Thus we might imagine Horatio to address Hamlet: "If my name is Horatio and I speak truth, it was thy father's ghost."

And Hamlet to reply: "Thy name is Horatio, but 'twas no ghost; ergo, thou speakest falsely."

The corresponding fallacy may be termed the **fallacy of denial of a premise.** A bad argument would result, if one were to reason as follows:

"The Germans may win on land, but if the British navy remains intact, they will not be able to dictate the peace. Therefore, if they win both on land and on sea, they will be able to dictate the peace."

COMPLEX HYPOTHETICAL SYLLOGISM

There is a further form of the hypothetical syllogism, to which attention may be profitably directed, but which is not specifically mentioned, I believe, in the elementary compendiums. We shall term it the **complex hypothetical syllogism.** Its general expression is:

> If x and y are true, then z is true;
> but z is untrue;
> therefore, either x is untrue or y is untrue.

The corresponding fallacy, or the one most commonly to be met with in connection with this argument, will be illustrated by a single example. It consists in interchanging minor and conclusion in the form given above.

"If all Shakespearean scholars are Englishmen and Churchill is a Shakespearean scholar, then Churchill is an Englishman; but either not all

Shakespearean scholars are Englishmen, or Churchill is not a Shakespearean scholar; therefore, Churchill is not an Englishman."

Another case would arise, if we were to infer, because the two premises of a true syllogism are false, that the conclusion is false.

Exercises

Select from the following such as are valid arguments:

1. If a thing can be conceived as nonexistent, its essence does not involve existence; but it is not of the essence of a centaur to exist; therefore, a centaur can be conceived as nonexistent.
2. The world cannot have existed always if it had a beginning in time; but it can have existed always; therefore, it had no beginning in time.
3. There would be no such thing as freedom, if all causation were according to natural law; but human freedom is a fact; therefore, not all causation is in accordance with natural law.
4. Human freedom is meaningless if God is omniscient, for he is then aware of what our decision in any case will be, before we decide; but man *is* a free agent; therefore, it cannot be that God is altogether omniscient.

CHAPTER XII

THE DILEMMA

The **dilemma** is an argument very commonly employed whenever it can be shown that any one of a number of possibilities leads to the same result. We shall introduce our theory of this mode of reasoning with a quotation from Archbishop Whately. It was "urged by the opponents of Don Carlos, the pretender to the Spanish throne; which he claimed as male heir, against his niece the queen, by virtue of the Salic law excluding females; which was established (contrary to the ancient Spanish usage) by a former king of Spain, and was repealed by King Ferdinand. They say 'if a king of Spain has a right to alter the law of succession, Carlos has no claim: and if no king of Spain has that right, Carlos has no claim; but a king of Spain either has or has not such right; therefore (on either supposition) Carlos has no claim.'" It will be a good exercise for the student, when he has finished his reading of this chapter, to undertake to retort this argument.

THE PARADOX OF GORGIAS

Another example is to be found in the contention of the Greek Gorgias that nature does not exist. Thus, if the world had a beginning in time, an in-

finite time must have elapsed before the moment of creation; but an infinite time never can elapse, and hence the moment of creation could never have arrived. Accordingly, the world is *uncreated.* If the world had no beginning in time (*i.e.,* has always existed), an infinite time must have elapsed before the present moment; but this is impossible; we cannot come to the end of something that has no end; therefore, the world must have been *created.*

The fact that time is infinite would have to be established by a separate proof. Thus, if past time were only finite, there must have been a *time* when there was no time (a formal contradiction); and if future time were only finite, there will come a *time* when there is no time (a formal contradiction). Consider the statement: x is both a and not-a, where x is some individual thing or class and a is a property, which it may or may not possess. It is evident that, of classes, the only values of x that will make this a true statement are classes that contain no objects. It will be untrue that triangles are both three-sided and not three-sided, but true that square-triangles are both square and not-square at the same time. Similarly, if x stands for *nothing,* the statement, nothing is both a and not-a, is true. Now, as regards the word, it is forced upon us that it is both *created* and *uncreated;* and since this is true, it is forced upon us that the world is *nothing* (*i.e.,* a nonexistent something).

THE PARADOX OF CORAX AND TISIAS

For the benefit of those who will refuse to consider seriously a philosophical dispute of this sort (and there are many such) we shall furnish an instance related to the experience of daily life. Before there was any science of logic, the Greek sophists (the traveling teachers) taught an art of argumentation, which was often applied practically within the courts of law. We cite the following well-known case from the *Lectures* of Sir William Hamilton: It is known as the *Litigiosus* or *Reciprocus*.

"Of the history of this famous dilemma there are two accounts, the Greek and the Roman. The Roman account is given us by Aulus Gellius, and is there told in relation to an action between Protagoras, the prince of the sophists, and Euathlus, a young man, his disciple. The disciple had covenanted to give his master a large sum to accomplish him as a legal rhetorician; the one half of the sum was paid down, and the other was to be paid on the day when Euathlus should plead and gain his first cause. But when the scholar, after the due course of preparatory instruction, was not in the same hurry to commence pleader as the master to obtain the remainder of his fee, Protagoras brought Euathlus into court and addressed his opponent in the following reasoning: "Learn, most foolish of

young men, that however matters may turn up (whether the decision to-day be in your favor or against you), pay me my demand you must. For if the judgment be against you, I shall obtain the fee by decree of the court, and if in your favor, I shall obtain it in terms of the compact, by which it became due on the very day you gained your first cause. You thus must fail, either by judgment or by stipulation." To this Euathlus rejoined: "Most sapient of masters, learn from your own argument that whatever may be the finding of the court, absolved I must be from any claim by you. For if the decision be favorable, I pay nothing by the sentence of the judges, but if unfavorable, I pay nothing in virtue of the compact, because, though pleading, I shall not have gained my cause." The judges, says Gellius, unable to find a *ratio decidendi,* adjourned the case to an indefinite day, and ultimately left it undetermined. I find a parallel story told, among the Greek writers, by Arsenius, by the Scoliast of Hermogenes, and by Suidas, of the rhetorician Corax (*anglicè* Crow) and his scholar Tisias. In this case, the judges got off by delivering a joke against both parties, instead of a decision in favor of either. We have here, they said, the plaguy egg of a plaguy crow, and from this circumstance is said to have originated the Greek proverb, *κακοῦ κόρακος κακὸν ᾠόν.*"

COMPLEX CONSTRUCTIVE DILEMMA

The **complex constructive dilemma** has the form: If x is true, then y is true, and if z is true, then w is true; but either x is true or z is true; therefore either y is true or w is true.

Our illustrations are taken from Whately. Thus, "If the obedience due from Subjects to Rulers extends to religious worship, the ancient Christians are to be censured for refusing to worship the heathen idols; if the obedience, etc., does not so extend, no man ought to suffer civil penalties on account of his religion; but the obedience, etc., either does so extend, or it does not; hence, either the ancient Christians are to be censured, etc., or else no man ought to suffer civil penalties on account of his religion.

Or, again: "If man is capable of rising, unassisted, from a savage to a civilized state, some instances may be produced of a race of savages having thus civilized themselves; and if man is not capable of this, then the first rudiments of civilization must have originally come from a superhuman instructor; but either man is thus capable, or not; therefore, either some such instance can be produced, or the first rudiments, etc."

COMPLEX DESTRUCTIVE DILEMMA

The **complex destructive dilemma** has the form: If x is true, then y is true, and if z is true, then w

is true; but either y is untrue or w is untrue; therefore, either x is untrue or z is untrue.

Thus: "If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest; but he does it either in jest or in earnest; therefore, he is either not wise, or not good."

The commonest fallacy to be noticed in connection with the dilemma is the case in which the minor is not a true disjunction—*i.e.*, the case in which the minor does not exhaust all of the possibilities. The conclusion can only be asserted when the premises are true, unless, to be sure, it is true in independence of the argument. We quote from Jevons:

"Dilemmatic arguments are, however, more often fallacious than not, because it is seldom possible to find instances where two alternatives exhaust all the possible cases, unless one of them be the simple negative of the other in accordance with the law of excluded middle. Thus if we were to argue that if a pupil is fond of learning he needs no stimulus, and that if he dislikes learning no stimulus will be of any avail, but as he is either fond of learning or dislikes it, a stimulus is either needless or of no avail, we evidently assume improperly the disjunctive minor premise. Fondness and dislike are not the only two possible alternatives, for there may be some who are neither fond of learning nor dislike it,

and to these a stimulus in the shape of rewards may be desirable."

A dilemma is said to be retorted, whenever an equally cogent dilemma to the contrary effect is produced. The retort of Euathlus to Protagoras is a case in point, and the argument of Gorgias contains two cogent dilemmas in the opposite sense, both of which the arguer accepts as valid. An Athenian mother, according to Aristotle, addressed her son in the following words: "Do not enter into public business, for if you say what is just, men will hate you; and if you say what is unjust, the gods will hate you." To which Aristotle retorts: "I ought to enter into public affairs; for if I say what is just, the gods will love me; and if I say what is unjust, men will love me."

PROOF OF THE DILEMMATIC ARGUMENT

In concluding this chapter we shall establish the formal validity of the dilemmatic argument, basing our proof on the principles which follow, and assuming (as is commonly done) the identity of the implication, x is untrue implies y is true and the disjunction, either x is true or y is true.

Principle I.—If antecedent and consequent of a valid implication be contradicted and interchanged, a valid implication will result.

Principle II.—If in any valid implication a factor

in the antecedent be strengthened, or the consequent be weakened, a valid implication will result.

Principle III.—If in any valid implication the same factor be conjoined to both antecedent and consequent, a valid implication will result.

(1) If (major) y is untrue implies x is untrue; and (minor) x is untrue implies z is true; then (conclusion) y is untrue implies z is true.
For (Principle II) the minor allows us to weaken the consequent of the major.

(2) If (antecedent) x is true implies y is true; then (consequent) y is untrue implies x is untrue.
For (Principle I) the antecedent and consequent of the principal antecedent may be contradicted and interchanged.

(3) If (major) x is true implies y is true; and (minor) x is untrue implies z is true; then (conclusion) y is untrue implies z is true.
For the major (1), being the same as the consequent of (2), may be strengthened (Principle II) to the antecedent of (2).

(4) If x is true implies y is true; and z is true implies w is true; and x is untrue implies z is true; then (consequent) y is untrue implies z is true and z is true implies w is true.
For (Principle III) the same factor (z is

true implies w is true) may be conjoined to both antecedent and consequent of (3).

(5) If (major) y is untrue implies z is true; and (minor) z is true implies w is true; then (conclusion) y is untrue implies w is true.
For (Principle II) the minor allows us to weaken the consequent of the major.

(6) If x is true implies y is true; and z is true implies w is true; and x is untrue implies z is true; then y is untrue implies w is true.
For the consequent of (4) being the same as the antecedent of (5) may be weakened (principle II) to the consequent of (5).

(7) If x is true implies y is true; and z is true implies w is true; and either x is true or z is true; then either y is true or w is true.
For we shall assume the right (as is commonly done) to replace the last premise and the conclusion of (6) by the disjunctions that appear in (7).

The result (7) will be recognized as the complex constructive dilemma, which has, accordingly, been derived from the three principles, which we assumed at the outset. The proof is laborious, when expressed without the aid of symbols. In that case the whole derivation could be set forth in a few lines. The proof of the complex destructive dilemma is left as an exercise for the student.

THE DILEMMA

Exercises

Select from the following such as are valid arguments:

1. If this man is a realist, he believes that the order of nature is independent of mind, and if he is an idealist and believes it dependent on mind, he must assume an absolute object; but either he believes it dependent or not; therefore in either case, he must profess belief either in an absolute object or in an absolute order of nature.
2. If a body moves, it must move in the place where it is or in the place where it is not; it cannot move where it is, for then it would not be there, and obviously it cannot move where it is not. Accordingly, its motion is impossible.

CHAPTER XIII

THE SORITES

The **sorites** is of the same form as the syllogism, for its terms are arranged in the same way in a cyclical order, but it is of a more general character. The number of terms is greater than three and, as in the case of immediate inference and syllogism, the number of premises is one less than the number of terms. Because there is sometimes a large number of premises, it will be more convenient to employ in place of the class-symbols, *a, b, c,* etc., the ordinal numbers, *1, 2, 3,* etc.

CONSTRUCTION OF THE VALID MOODS

We shall begin by illustrating the manner of constructing a valid mood of the sorites from a chain of valid syllogisms. Suppose that we were to be given the chain,

A(*21*) and A(*32*) implies A(*31*),
A(*31*) and A(*43*) implies A(*41*),
A(*41*) and A(*54*) implies A(*51*),

and were asked what valid moods of the sorites is thereby implied. It is clear that the major premise of the last syllogism, being the same as the con-

clusion of the second, may be strengthened to A(*31*) and A(*43*). The immediate result of this strengthening is a valid mood of the sorites, *viz.*,

A(*31*) and A(*43*) and A(*54*) implies A(*51*).

The major premise of this last implication may in turn be strengthened to A (*21*) and A (*32*) because of the first syllogism, and we should have

A(*21*) and A(*32*) and A(*43*) and A(*54*) implies A(*51*).

This, then, is the valid mood of the sorites whose truth is implied by the chain of generating syllogisms.

Another method of constructing a valid mood of the sorites from a chain of syllogisms depends upon another principle.

Principle.—If in any valid implication the same factor be conjoined to both antecedent and consequent, a valid implication will result.

Let our chain of syllogisms be

E(*21*) and A(*32*) implies E(*31*),
E(*31*) and I(*34*) implies O(*41*),
O(*41*) and A(*45*) implies O(*51*),

and suppose that we conjoin to antecedent and con-

sequent of the first syllogism the minor premise of the second and so obtain

E(*21*) and A(*32*) and I(*34*) implies
E(*31*) and I(*34*).

The second syllogism allows us to weaken the consequent of this result to O(*41*) and upon carrying out this operation we should obtain

E(*21*) and A(*32*) and I(*34*) implies O(*41*).

Now conjoin to antecedent and consequent of this mood of the sorites the minor premise of the third syllogism, A(*45*)—that is,

E(*21*) and A(*32*) and I(*34*)
and A(*45*) implies O(*41*) and A(*45*),

and weaken the consequent of this implication to O(*51*) by authority of the last member of the chain. Accordingly,

E(*21*) and A(*32*) and I(*34*)
and A(*45*) implies O(*51*),

is the valid mood of the sorites which was to be generated.

THE INVERSE OPERATION

Finally, let us consider the operation which is inverse to the preceding. Suppose, being given a valid mood of the sorites, we should be asked to find

the chain of syllogisms upon which it depends. Let the mood be

$$A(12) \text{ and } A(23) \text{ and } O(43) \text{ and } A(45) \text{ and } A(56) \text{ implies } O(61).$$

The premises of the first syllogism of the chain will be the same as the first two premises of the sorites and the minor of the second syllogism will be the same as the third premise of the sorites and so on. The fragment of the chain so far ascertained will be:

$$\begin{array}{l} A(12) \text{ and } A(23) \text{ implies} \\ \qquad\quad \text{and } O(43) \text{ implies} \\ \qquad\quad \text{and } A(45) \text{ implies} \\ \qquad\quad \text{and } A(56) \text{ implies} \end{array}$$

Now the conclusion of the first syllogism, whose premises appear out of the normal order, is evidently $A(13)$ and this must be the major of the syllogism which follows. The conclusion of this in turn is determined as $O(41)$. If we were to continue this process, each member of the chain in succession would be ambiguously determined as,

$$\begin{array}{l} A(12) \text{ and } A(23) \text{ implies } A(13), \\ A(13) \text{ and } O(43) \text{ implies } O(41), \\ O(41) \text{ and } A(45) \text{ implies } O(51), \\ O(51) \text{ and } A(56) \text{ implies } O(61). \end{array}$$

RULES OF THE SORITES

From the considerations that have gone before we conclude that certain valid moods of the sorites can be constructed from chains of valid syllogisms. A complete solution of the sorites would contain a proof that the only valid moods that exist can be built up from chains of valid syllogisms in the manner described. This proof is too advanced for a work of this character. We observe, however, that it depends upon the following truths:

1. If the conclusion is affirmative, then all the premises are affirmative.
2. If the conclusion is negative, then one and only one premise is negative.
3. If the conclusion is universal, then all the premises are universal.
4. If the conclusion is particular, then not more than one premise is particular.

Exercises

1. What valid mood of the sorites can be generated from the following chain of syllogisms?

$$\begin{array}{c} A(21) \text{ and } A(32) \text{ implies } A(31), \\ A(31) \text{ and } A(43) \text{ implies } A(41), \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ A(n\text{-}1\ 1) \text{ and } A(n\ n\text{-}1) \text{ implies } A(n\ 1). \end{array}$$

2. From what chain of valid syllogisms can we build up the sorites,

$A(21)$ and $A(32)$ and . . .
. . . and $A(t\ t\text{-}1)$ and $I(t\ t+1)$ and $A(t+1\ t+2)$ . . .
. . . and $A(n\text{-}1\ n)$ implies $I(n\ 1)$?

3. Which one of the rules enunciated at the end of this chapter will establish the invalidity of the sorites,

$$E(12) \text{ and } E(23) \text{ and } E(34) \text{ implies } E(41)?$$

4. State a rule analogous to one of the rules of the syllogism which will declare to be invalid:

$$A(12) \text{ and } A(32) \text{ and } A(43) \text{ implies } A(41).$$

CHAPTER XIV

THE VERIFICATION OF THE CLASSICAL SYSTEM

In order that the classical system should hold true in all of its parts, certain characteristic conditions must be satisfied. Besides the moods *Barbara* and *Celarent,* which have been set down as postulates, we should have to have:

(1) Corresponding to each member of the set, A, E, I, O, there is another member of the same set which stands for its contradictory.

(2) Subalternation, A implies I, and E implies O, holds true.

(3) The subject and predicate of E and I alone are simply convertible.

SUPPOSED BREAKDOWN OF THE CLASSICAL SYSTEM

To-day it is all but universally taken for granted that not all these conditions hold for all meanings of the terms, and it is usual to retain the first and last and to assert that the relation of subalternation breaks down. Citations similar to the two set down below might have been selected from any one of a dozen separate authors. Thus Mr. Bertrand Russell remarks:[1] "With our definitions, All S is P

[1] *Introduction to Mathematical Philosophy,* p. 164. London, 1919.

does not imply Some S is P, since the first allows the nonexistence of S and the second does not; thus conversion *per accidens* becomes invalid, and some of the moods of the syllogism are fallacious—*e.g., Darapti:* All M is S, All M is P, therefore Some S is P, which fails if there is no M." Or, to quote from Padoa's exposition of Peano's logic[1]: "The untruth of the traditional moods of the syllogism, by means of which from two *universal* judgments one would deduce a *particular* judgment, has been recognized separately by Miss Ladd (1883), Schröder, Nagy, Peano, etc. It is one of the first and most remarkable results of the adoption of a logical ideography."

In order to understand how this breakdown occurs, the student must become acquainted with what is termed in modern logical theory a *null*-class—that is, a class which contains no objects, or whose value in extension is zero. Thus, the class "square circles" is null and so is the class "fractional integers." The implication,

> All square circles are circles implies
> that some square circles are circles,

is supposed to break down, because the antecedent is taken to be true and the consequent is taken

[1] "*La Logique Déductive,*" etc., *Revue de Métaphysique et de Morale,* vol. 20, p. 67.

to be false; it being commonly assumed that the particular affirmative, some *a* is *b*, is untrue unless members of the *ab* class (what is *a* and *b*) exist.

To consider the case of a syllogism that is supposed to be fallacious let us select the mood *Darapti*,

All square circles are circles,
All square circles are squares,
Some squares are circles,

which appears to be invalid if the premises are taken to be true.

REASON FOR THIS MISAPPREHENSION

This apparent breakdown of the classical system depends, however, upon the fact that the interpretation of the Aristotelian propositions has been too narrowly rendered. Thus, it has seemed altogether natural to the workers in this field to recognize the following identities:

$$\begin{aligned} \mathrm{A}(ab) &= a \text{ is included in } b, \\ \mathrm{E}(ab) &= a \text{ is included in non-}b, \\ \mathrm{I}(ab) &= a \text{ is not included in non-}b, \\ \mathrm{O}(ab) &= a \text{ is not included in } b. \end{aligned}$$

It will be observed that on this interpretation A and O, and E and I are contradictory pairs. Also it is

true that E and I alone are simply convertible; for, in the first place,

> If *a* is included in non-*b*
> then *b* is included in non-*a*,

is a true statement in the modern Boolean algebra and the corresponding implication holds for the case of I; and, secondly,

> If *a* is included in *b*,
> then *b* is included in *a*,

is untrue, and the same holds for the corresponding case of O.

Subalternation, however, breaks down, for if *a* and *b* are both *null*, and non-*a* and non-*b*, consequently, *universe*, we should have:

> If the members of the null-class are included among the members of the null-class, then the members of the null-class are not contained among the members of the universe.

Here the antecedent is true and the consequent is false, so that subalternation fails, when the terms are taken to represent empty classes.

SOLUTION OF THE DIFFICULTY

We shall now give a rendering of the propositional functions, A, E, I, and O, which not only will verify all of the implications of the classical logic set down in the preceding chapters, but which will accord equally well with the usage of language and with common sense.[1] These somewhat more extended meanings are:

A(ab) = a is included in b **and** a is not null **and** b is not null,
E(ab) = a is included in non-b, **or** a is null **or** b is null.
I(ab) = a is not included in non-b **and** is not null **and** b is not null.
O(ab) = a is not included in b **or** a is null **or** b is null.

If the student will recall what was said in Chapter III of the manner in which a conjunction may be expressed as a disjunction and conversely, he will have no difficulty to verify the fact that A and O, and that E and I are contradictory pairs. Moreover, it will be clear that E and I alone are simply convertible. It will prove a more difficult task for him to verify the relation of subalternation, but he may, perhaps, satisfy himself by considering the case in which the terms are allowed to become null.

[1] Professor Singer has suggested an interpretation of these forms which retains A(aa), but gives up E(aa'). No interpretation will conform *altogether* with common usage.

In the case of *Darapti* he will be able to see that the premises become false when the middle term becomes null and that, consequently, the mood remains valid under all conditions. If the reader happens to be conversant with the modern calculus of classes, and so habitually employs symbols to designate the operations of logic, he will be able easily to verify all of the postulates that have been set down in the chapters that have gone before.

RECONCILIATION WITH COMMON SENSE

The classical logic, therefore, does not break down, as is commonly supposed, but may be taken to hold true in all of its parts. It only remains to be shown that the interpretations of the propositions, A, E, I, and O, which we have given above, are as much in accord with common experience and with common sense as the ones that are ordinarily accepted. Consider the proposition "all *a* is *a*." On our interpretation this will be true for all meanings of *a* except when *a* is null. Thus,

All Athenians are Athenians

means, when expressed in full,

> The members of the class Athenians are included among the members of the class Athenians; and Athenians are not included among non-Athenians."

And this is a true statement.

There are cases, too, in which common sense agrees with the interpretations of the categorical forms that have just been given, and disagrees with those that are commonly recognized. Suppose the proposition:

> Every aged Achilles who is mentioned in the "Iliad" is there celebrated as the swift of foot.

Now, since only one Achilles is mentioned in the "Iliad" and since he died in his youth, the subject of this proposition is a null or an empty class. On the rendering usually given the above statement would be true; on the amended rendering suggested above it would be false, and it is certain that common sense is in agreement with the last interpretation and not with the former.

Again, the assertion, no *a* is non-*a*, is true by both renderings. On the first interpretation it becomes

> Athenians are included among non-non-Athenians,

and, on the second interpretation, it becomes the same with the added condition,

> Or else Athenians is a null-class,

it being allowed to omit this last part from the com-

plete disjunction, because it is (empirically) untrue.

EFFECT ON CONTRAPOSITION AND ON OBVERSION

It is appropriate to mention here some of the transformations that are recognized in traditional logic. The operation of conversion of the terms by *contraposition* as explained before, consists in replacing the terms by their negatives and interchanging them. It is supposed to hold of A and O, but not of E and I. Thus,

All *a* is *b*

is taken to be equivalent to

All non-*b* is non-*a*.

We observe that, except for the meanings *null* and *universe*—that is, for all concrete meanings of the terms but these—contraposition holds in the rendering of A and O which we have given. Thus,

All Athenians are Greeks

becomes as a result of this operation:

> Non-Greeks are included among non-Athenians; and non-Greeks are not Greeks; and non-Athenians are not Athenians,

a statement which is manifestly true, and

Some Greeks are not Athenians

becomes, when converted by contraposition,

> Non-Athenians are not included among non-Greeks; or else non-Greeks are Greeks; or else non-Athenians are Athenians.

the latter parts of the disjunction seeming in common sense rather to reinforce the truth of the part that is first expressed.

Again, except for the meanings *null* and *universe*—that is, for all other meanings of the terms—the identity commonly assumed for "no *a* is *b*" and "all *a* is non-*b*," will hold true. In general it may be observed that the interpretation which we have given of the propositions, A, E, I, and O are identical with the ones that are traditionally offered, for all meanings of the terms except for these limiting values. We conclude, then, that the classical logic may be considered to hold true in all of its parts and that the modern view, that subalternation and some of the valid moods of the syllogism are fallacious, is based upon a misapprehension.

BIBLIOGRAPHY

For the benefit of the student who may desire to acquaint himself with the more recent trend of logical research a few references in English are listed below. The work of Boole may furnish him a starting point, but in such a field there is no set manner of beginning, just as there is no royal road to the end. In German there is Schröder's *Abriss,* by Müller, Leipzig, Teubner, 1909; and in French, Padoa's introduction to Peano's system, Paris, Gauthier-Villars, 1912. Before he attempts the *Principia Mathematica* of Whitehead and Russell he will do well to consult the admirable *Survey* of Mr. C. I. Lewis. He will find there a bibliography that will be quite sufficient for all of his purposes.

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